Physical Processes in the Tokamak Edge/Pedestal

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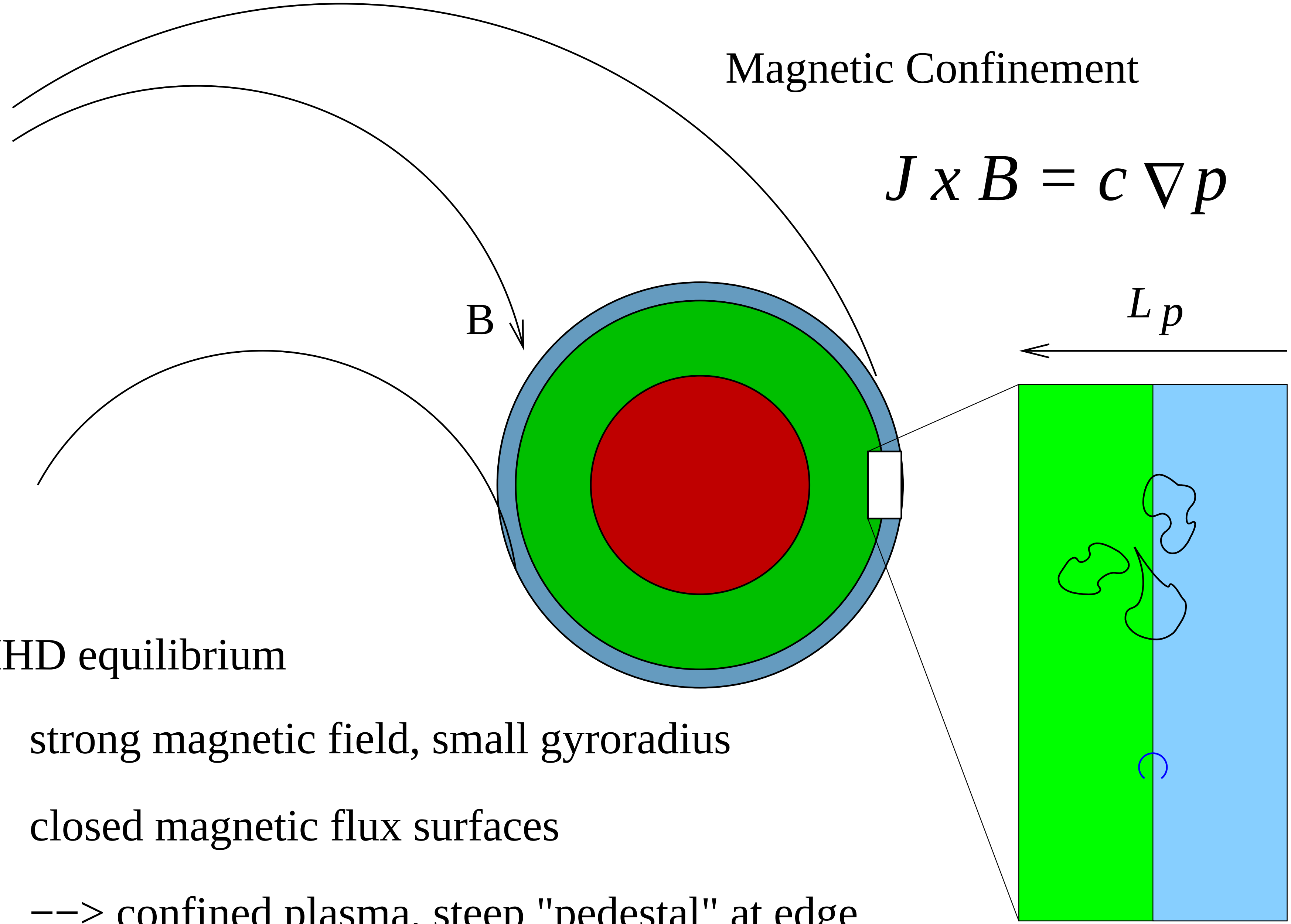
Magnetic Confinement

$$J \times B = c \nabla p$$



MHD equilibrium

strong magnetic field, small gyroradius

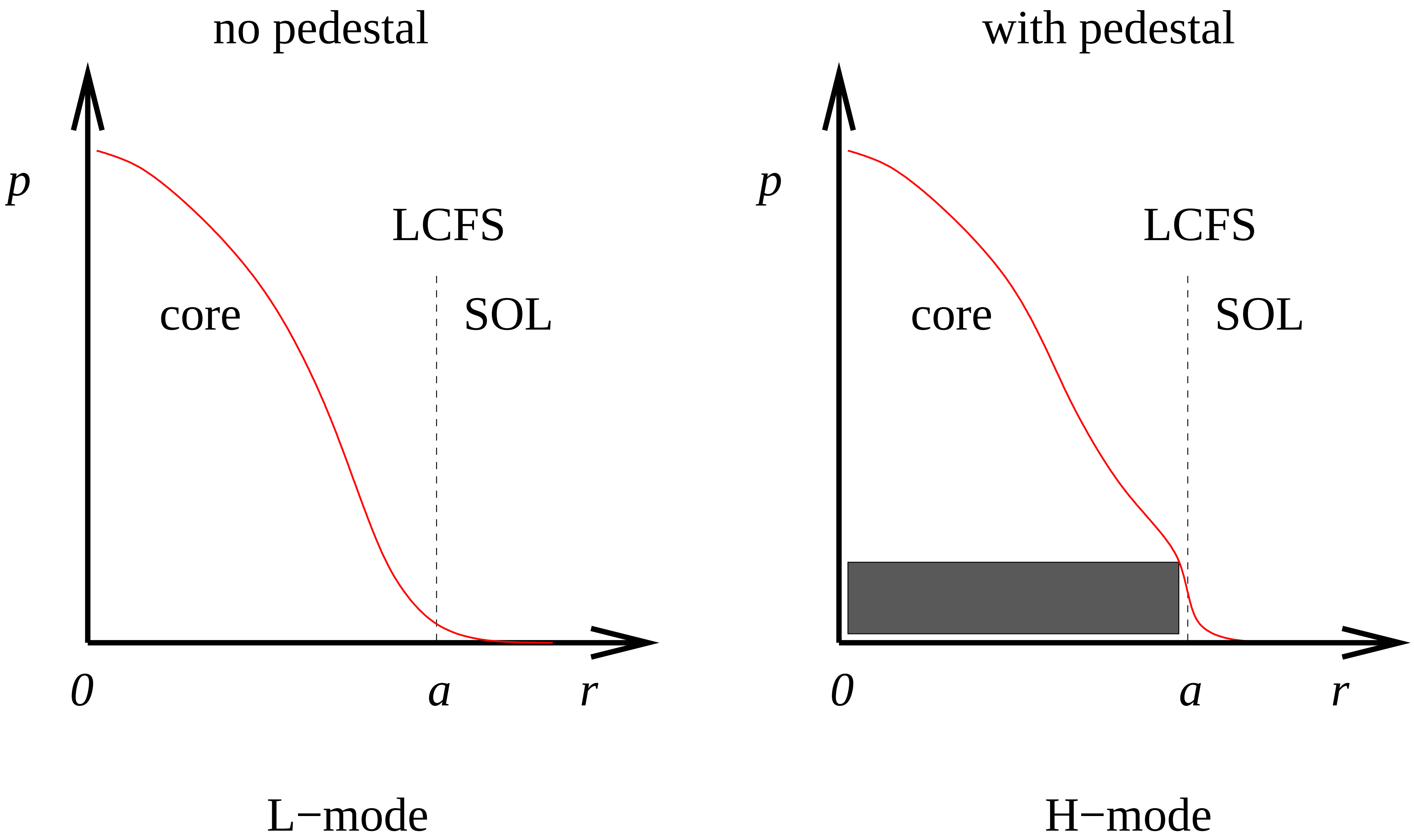
closed magnetic flux surfaces

−−> confined plasma, steep "pedestal" at edge

however . . .   turbulence, MHD events −−> losses

eddies, few gyroradii

Profile Examples



tokamak edge pedestal distinctly

- **scales – parameters**
  — steep gradient, $R/L_\perp > 30$, even find $L_\perp/R < \rho_s/L_\perp$
  — electron transit frequencies comparable to turbulence, $c_s/L_\perp > V_e/qR$

- **dynamics**
  — low frequency/beta $\rightarrow$ magnetic stiffness $\rightarrow$ reduced, gyrokinetic equations
  — electromagnetic turbulence character $c_s/L_\perp > v_A/qR$

- **equilibration**
  — neoclassical time scale separation is affected
  — collisional relaxation in the regime of $L_\perp/R < \rho_s/L_\perp \sim \{20, 30\}^{-1}$

the scale ratio regime determines
edge/pedestal dynamical character

space scales

- meaning of steep gradients:

$$\text{profile scale} \quad L_\perp \quad \ll \quad \text{toroidal major radius} \quad R$$

- field line pitch parameter in a conventional tokamak

$$q \qquad \sim \qquad 3$$

- parallel scale (field line connection length) is the largest

$$qR/L_\perp \quad \sim \quad 200$$

- the local rho-star is not smaller than the perp scale ratio

$$\rho_s/L_\perp \quad \sim \quad 1/30 \qquad\qquad L_\perp/R \quad \sim \quad 1/50$$

time scales

- transit frequencies for electrons, magnetic field
  - thermal or Alfvén velocity and parallel scale

- turbulence spectrum range, acoustic frequencies go with sound speed

- parallel scales have consequences for time scale separation

$$\frac{c_s}{L_\perp} \quad > \quad \frac{v_A}{qR} \quad \sim \quad \frac{V_e}{qR} \quad \gg \quad \frac{c_s}{R} \quad > \quad \frac{c_s}{qR}$$

- the space scale ratio affects relaxation

$$\text{several} \quad 10^2 \frac{L_\perp}{c_s} \quad \sim \quad \nu_i^{-1} \qquad \text{and in some cases} \qquad \text{several} \quad \nu_i^{-1} \quad \sim \quad \frac{L_\perp^2}{\chi_i}$$

what determines the edge?

- mainly, the electron thermal nonadiabaticity condition: $c_s/L_\perp > V_e/qR$, or $\hat{\mu} > 1$

$$\text{def:} \qquad \hat{\mu} = \frac{m_e}{M_D}\left(\frac{qR}{L_\perp}\right)^2$$

- consider the boundary, $\hat{\mu} = 1$, then $c_s/L_\perp = V_e/qR \quad \rightarrow \quad$ edge/core bndy
- solve this for the profile scale length

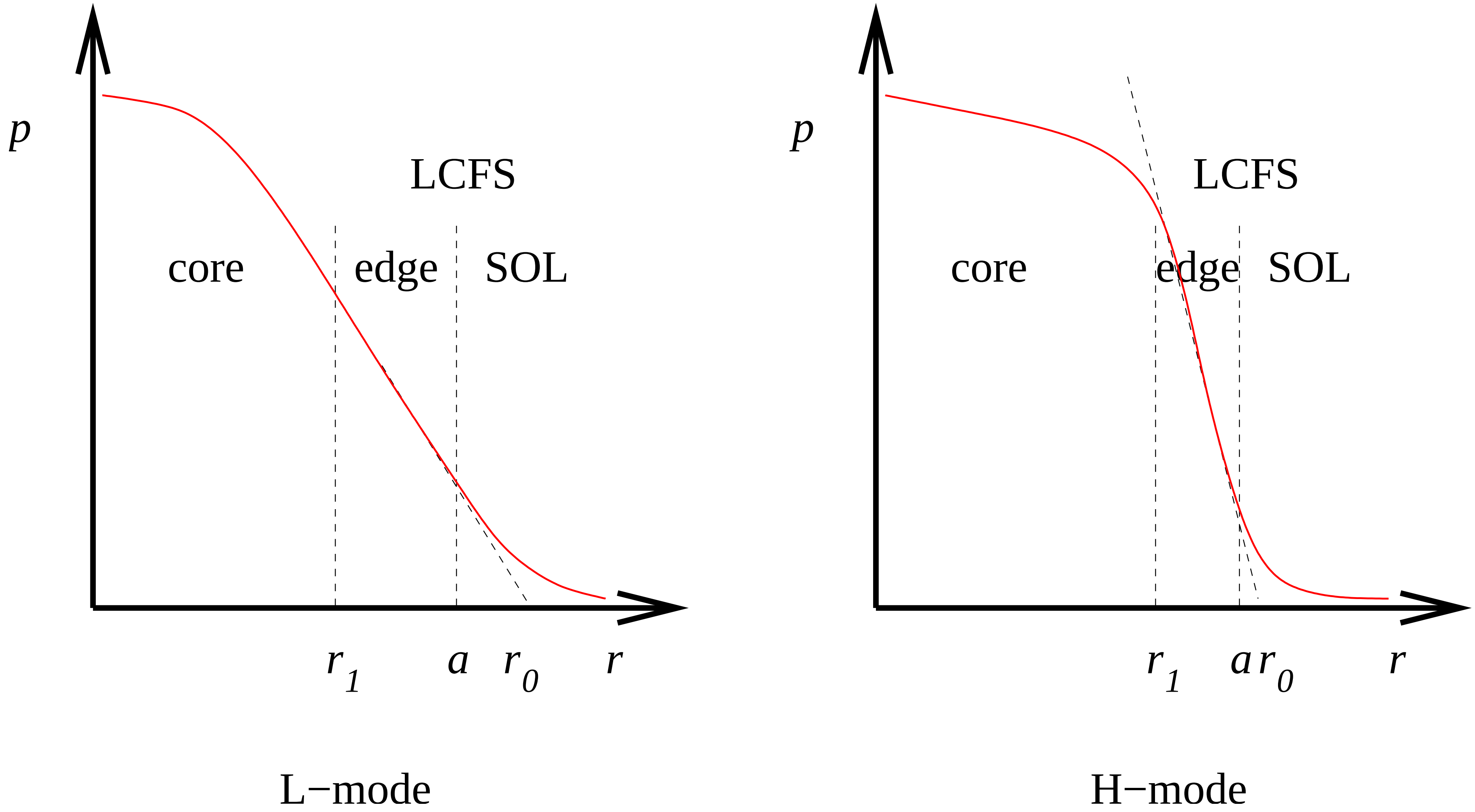
$$L_\perp = \sqrt{m_e/M_D}\, qR$$

- for linear profile gradients this is typically about 8 cm
  - and it holds over about the last 4 cm within the LCFS

on the other hand,
if a pedestal exists, the top is the edge/core boundary

Edge Layer Extent

$\hat{\mu} = 1$ at $r = r_1$



Edge Layer Extent

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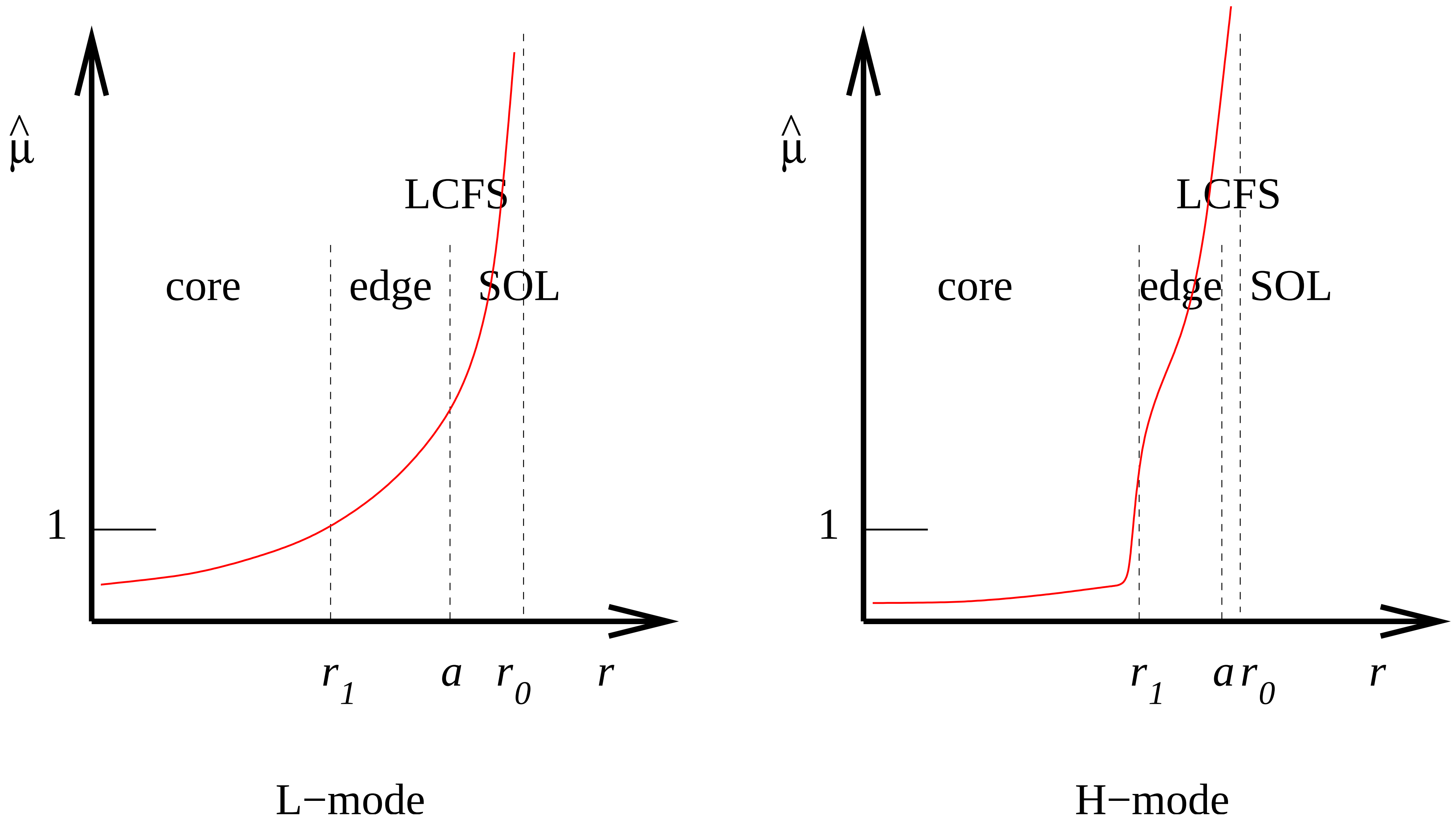


Low Pressure (Beta) Dynamics

low ''beta''

$p << B^2/8\pi$

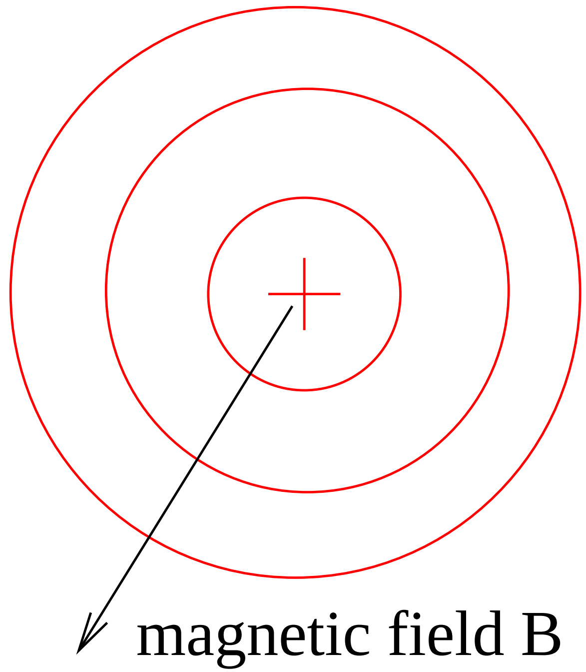
low frequencies

$\omega << k_\perp v_A$

''flute mode''

vortices/filaments

$k_{||} << k_\perp$



pressure disturbance $\tilde{p}$

magnetic disturbance $\tilde{B}$ (parallel to B)

magnetic field B

--> strict perpendicular force balance $\nabla(4\pi\,\tilde{p} + B\tilde{B})$

$\omega \sim k_{||} v_A$ --> electromagnetic parallel dynamics

low frequency drift regime

- fluid $\rightarrow$ reduced equations

$$\mathbf{E} \quad \rightarrow \quad -\frac{1}{c}\frac{\partial A_\parallel}{\partial t}\mathbf{b} - \nabla\phi \qquad\qquad \nabla\cdot\mathbf{u}_\perp \quad \sim \quad \frac{\mathbf{u}_\perp}{R} \quad \ll \quad \mathbf{u}\cdot\nabla$$

$$\mathbf{v}_E = \frac{c}{B^2}\mathbf{B}\times\nabla\phi$$

$$B^2 = \frac{B_0^2 R_0^2}{R^2}\left[1 + O\left(\frac{a}{qR}, \beta\right)\right] \quad \rightarrow \quad \frac{I^2}{R^2}$$

- kinetic $\rightarrow$ gyrokinetic

$$\mathbf{A} \quad \rightarrow \quad \mathbf{A}_{\rm eq} + A_\parallel\mathbf{b} \qquad\qquad \mathbf{b} \quad \rightarrow \quad R\nabla\varphi \qquad\qquad \psi_T \quad \rightarrow \quad \psi_{\rm eq} + A_\parallel R$$

def: $\Omega_E = \frac{c}{B}\nabla_\perp^2\phi \quad \ll \quad \frac{eB}{mc}$ $\qquad$ $\mu = \frac{mv_\perp^2}{2B} \quad \rightarrow$ conserved, i.e., $\frac{d\mu}{dt} = 0$

then: $f = f(\mathbf{R}, z, \mu, t)$ but dynamics (exc. coll.) is in space of $\{\mathbf{R}, z\}$

example – magnetic compressibility

- write MHD equations for $\mathbf{u}$ and $\mathbf{B}$

$$\rho_M \frac{d\mathbf{u}}{dt} = \frac{1}{c}\mathbf{J}\times\mathbf{B} - \nabla p \qquad\qquad \frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B})$$

- in tokamak geometry, $\mathbf{B} = F_{\rm dia}\nabla\varphi + \nabla\psi_T\times\nabla\varphi,$ and in general: $\mu_0\mathbf{J} = \nabla\times\mathbf{B}$
- form equations for $\nabla\cdot\mathbf{u}$ and $F_{\rm dia}$

$$\nabla\cdot\mu_0\rho_M R^2 \frac{d\mathbf{u}}{dt} = -\nabla^2 \frac{F_{\rm dia}^2}{2} - \nabla\cdot\left(\Delta^*\psi_T\nabla\psi_T + \mu_0 R^2\nabla p\right)$$

$$\frac{\partial}{\partial t}\frac{F_{\rm dia}}{R^2} = -F_{\rm dia}\nabla\cdot\frac{\mathbf{u}}{R^2} - \frac{\mathbf{u}}{R^2}\cdot\nabla F_{\rm dia} + \mathbf{B}\cdot\nabla\left(\mathbf{u}\cdot\nabla\varphi\right)$$

- for $k_\perp v_A \quad\gg\quad \partial/\partial t$ and reasonable velocities these reduce to

$$F_{\rm dia} \quad\rightarrow\quad I = \text{constant} \qquad\qquad \nabla\cdot\frac{\mathbf{u}}{R^2} \quad\rightarrow\quad 0 \quad\rightarrow\quad \text{drifts}$$

drifts in reduced model

- solve for $\mathbf{u}$ in Lorentz force instead of inertia since $ZeB/mc \gg \partial/\partial t$

$$nm\frac{d\mathbf{u}}{dt} + \nabla\cdot\mathbf{\Pi}^* = nZe\left(\mathbf{E} + \frac{\mathbf{u}}{c}\times\mathbf{B}\right) - \nabla p$$

- find drifts

$$\mathbf{u}_\perp = \frac{c}{B^2}\mathbf{B}\times\left(\nabla\phi + \frac{1}{nZe}\nabla p\right) + \frac{mc}{ZeB^2}\mathbf{B}\times\left(\frac{d\mathbf{u}}{dt} + \frac{1}{nm}\nabla\cdot\mathbf{\Pi}^*\right)$$

- last term is polarisation velocity, and total inertia $\rightarrow$ polarisation current

all inertia is polarisation (incl. diamag. momentum flux)

vorticity in reduced model

- vorticity instead of fluid inertia (all inertia becomes polarisation)

$$\frac{\partial \varpi}{\partial t} + [\phi, \varpi] = B\nabla_\parallel \frac{J_\parallel}{B} - \mathcal{K}(p) \qquad \leftrightarrow \qquad \nabla\cdot\mathbf{J} = 0$$

- geometry and ordering under $L_\perp \ll R$ and $nmu_E^2 \ll p$ and $\rho_s \ll L_\perp$

$$[f,g] = \frac{cR^0}{B^0}\nabla\varphi\cdot(\nabla f\times\nabla g) \qquad B\nabla_\parallel f = \mathbf{B}\cdot\nabla f = \mathbf{B}^0\cdot\nabla f - \nabla\varphi\cdot\left(\nabla A_\parallel R\times\nabla f\right)$$

$$\mathcal{K}(f) = [(R/R_0)^2,\ f] \qquad \nabla\cdot f\mathbf{v}_E \quad\rightarrow\quad [\phi, f] - f_0\mathcal{K}(\phi)$$

- vorticity in the reduced MHD limit

$$\varpi = \nabla\cdot\frac{\rho_M c^2}{B^2}\nabla_\perp\phi \quad\rightarrow\quad \frac{\rho_{M0}c^2}{B_0^2}\nabla_\perp^2\phi \quad\rightarrow \text{normalisation} \rightarrow\quad \rho_s^2\nabla_\perp^2\phi$$

normalisation and scales

- low-freq, pressure driven $\quad\rightarrow\quad$ sound speed normalisation, not an Alfvénic one

$$n \leftrightarrow n_0 \qquad \phi \leftrightarrow \frac{T_e}{e} \qquad u_\parallel \leftrightarrow c_s \qquad J_\parallel \leftrightarrow n_0 e c_s \qquad A_\parallel \leftrightarrow B_0 \rho_s$$

- for vorticity, divide apparent charge density by $n_0 e$, use above, to find

$$\varpi \leftrightarrow \frac{\rho_{M0} c^2}{B_0^2} \frac{T_e}{n_0 e^2} \nabla_\perp^2 \phi \rightarrow \rho_s^2 \nabla_\perp^2 \phi \qquad \rho_s^2 = \frac{c^2 M_i T_e}{e^2 B_0^2} \quad \rightarrow \quad \rho_s = \frac{c_s}{\Omega_i}$$

- the scale ρ_s is demanded by eventual $n_e e \nabla_\parallel \phi \sim \nabla_\parallel p_e \quad\rightarrow\quad T_e/e$ for ϕ
  - this is the main neglect done by the MHD model

- time scale is nominal space scale divided by c_s, so using $L_\perp$ you have

$$t \leftrightarrow \frac{L_\perp}{c_s} \qquad \delta = \frac{\rho_s}{L_\perp} \qquad \text{and} \quad \mathcal{K}, \quad [f,g] \rightarrow O(\delta) \quad \text{while} \quad \rho_s^2 \nabla_\perp^2 \rightarrow O(\delta^2)$$

MHD dynamics in reduced model

- in one-fluid MHD (familiar) you neglect diamagnetic effects
  - all appearances of ∇p next to $\nabla\phi$
  - implicitly assumes smallness of pressure fluctuations/dynamics

- solve for vorticity, electron density, and electron and ion parallel dynamics
  - any model used for learning purposes is isothermal

$$\frac{\partial\varpi}{\partial t} + [\phi, \varpi] = B\nabla_\parallel \frac{J_\parallel}{B} - \mathcal{K}(n)$$

$$\frac{\partial n}{\partial t} + [\phi, n] + B\nabla_\parallel \frac{u_\parallel}{B} = \mathcal{K}(\phi)$$

$$\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel\phi - \eta_\parallel J_\parallel$$

$$\frac{\partial u_\parallel}{\partial t} + [\phi, u_\parallel] = -\nabla_\parallel n + \mu_\parallel \nabla_\parallel^2 u_\parallel$$

- with $\varpi = \rho_s^2\nabla_\perp^2\phi$ and $J_\parallel = -(\rho_s^2/\beta_e)\nabla_\perp^2 A_\parallel$ as self-consistent field equations

Reynolds stress and acoustic oscillations

- Reynolds stress is the same as polarisation nonlinearity

$$\frac{\partial\varpi}{\partial t} + \mathbf{v}_E\cdot\nabla\varpi = \cdots \quad \text{zonal component} \quad \rightarrow \quad \frac{\partial}{\partial t}\left\langle\varpi\right\rangle = \frac{\rho_s}{L_\perp}\rho_s^2\frac{\partial^2}{\partial x^2}\left\langle\frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial y}\right\rangle + \cdots$$

- zonal flow energy (grows if stress is aligned to zonal vorticity)

$$-\left\langle\phi\right\rangle\frac{\partial}{\partial t}\left\langle\varpi\right\rangle = \nabla\cdot(\cdots) + \frac{\rho_s}{L_\perp}\left\langle\varpi\right\rangle\left\langle-\frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial y}\right\rangle + \cdots$$

- toroidicity gives rise to geodesic acoustic oscillation
  - take zonal component, find mode frequency
  - factors of $\rho_s\partial/\partial x$ cancel, and the 1/2 is the average of $\sin^2\theta$
  - thermal dynamics add coefficients, ion parallel dynamics adds corrections

$$\frac{\partial\varpi}{\partial t} + \cdots = -\mathcal{K}(n) \qquad\qquad \frac{\partial n}{\partial t} + \cdots = \mathcal{K}(\phi) \quad \rightarrow \quad \frac{\partial^2}{\partial t^2} = -\frac{1}{2}\left(\frac{2c_s}{R}\right)^2$$

new things at the fluid level

- in edge turbulence or any pedestal dynamics, never neglect ∇p against $\nabla\phi$
  ◦ this makes everything two-fluid, at least

- adiabatic coupling

$$\text{dynamics} \qquad \widetilde{p}_e \quad \leftrightarrow \quad J_\| \quad \leftrightarrow \quad \widetilde{\phi}$$

$$\text{sidebands} \qquad \langle p_e \sin\theta \rangle \quad \leftrightarrow \quad \left\langle J_\| \cos\theta \right\rangle \quad \leftrightarrow \quad \langle \phi \sin\theta \rangle$$

- diamagnetic compression

$$\text{sidebands} \qquad \langle p \sin\theta \rangle \quad \leftrightarrow \quad \langle p \rangle$$

- MHD and acoustic systems coupled by both processes

- flow dynamics in simple models drastically altered

dynamics in reduced two-fluid model

- now you never neglect diamagnetic effects
  - pressure dynamics is never "small" compared to flows, currents
- solve for vorticity, electron density, and electron and ion parallel dynamics
  - any model used for learning purposes is isothermal
  - write density as isothermal p_e to remember the physics, write normed masses $\mu_{i,e}$

$$\frac{\partial \varpi}{\partial t} + [\phi, \varpi] = B\nabla_\parallel \frac{J_\parallel}{B} - \mathcal{K}(p_e)$$

$$\frac{\partial p_e}{\partial t} + [\phi, p_e] + B\nabla_\parallel \frac{u_\parallel - J_\parallel}{B} = \mathcal{K}(\phi - p_e)$$

$$\frac{\partial}{\partial t}\left(A_\parallel + \mu_e J_\parallel\right) + [\phi, \mu_e J_\parallel] = \nabla_\parallel (p_e - \phi) - 0.51\mu_e \nu_e J_\parallel$$

$$\mu_i \frac{\partial u_\parallel}{\partial t} + [\phi, \mu_i u_\parallel] = -\nabla_\parallel p_e + \mu_\parallel \nabla_\parallel^2 u_\parallel$$

- with $\varpi = \rho_s^2 \nabla_\perp^2 \phi$ and $J_\parallel = -(\rho_s^2/\beta_e)\nabla_\perp^2 A_\parallel$ as self-consistent field equations

turbulence energetics

- adiabatic response allows ExB and thermal coupling through $J_\parallel$
  - as well as the curvature coupling that exists in one-fluid model

- free energy construction – identify transfer effects as pieces of total divergences
  - multiply by $-\phi$ and $J_\parallel$ and p_e respectively

$$\frac{1}{2}\frac{\partial}{\partial t}\left|\rho_s\nabla_\perp\phi\right|^2 + \nabla\cdot() = -\phi B\nabla_\parallel\frac{J_\parallel}{B} + \phi\mathcal{K}(p_e)$$

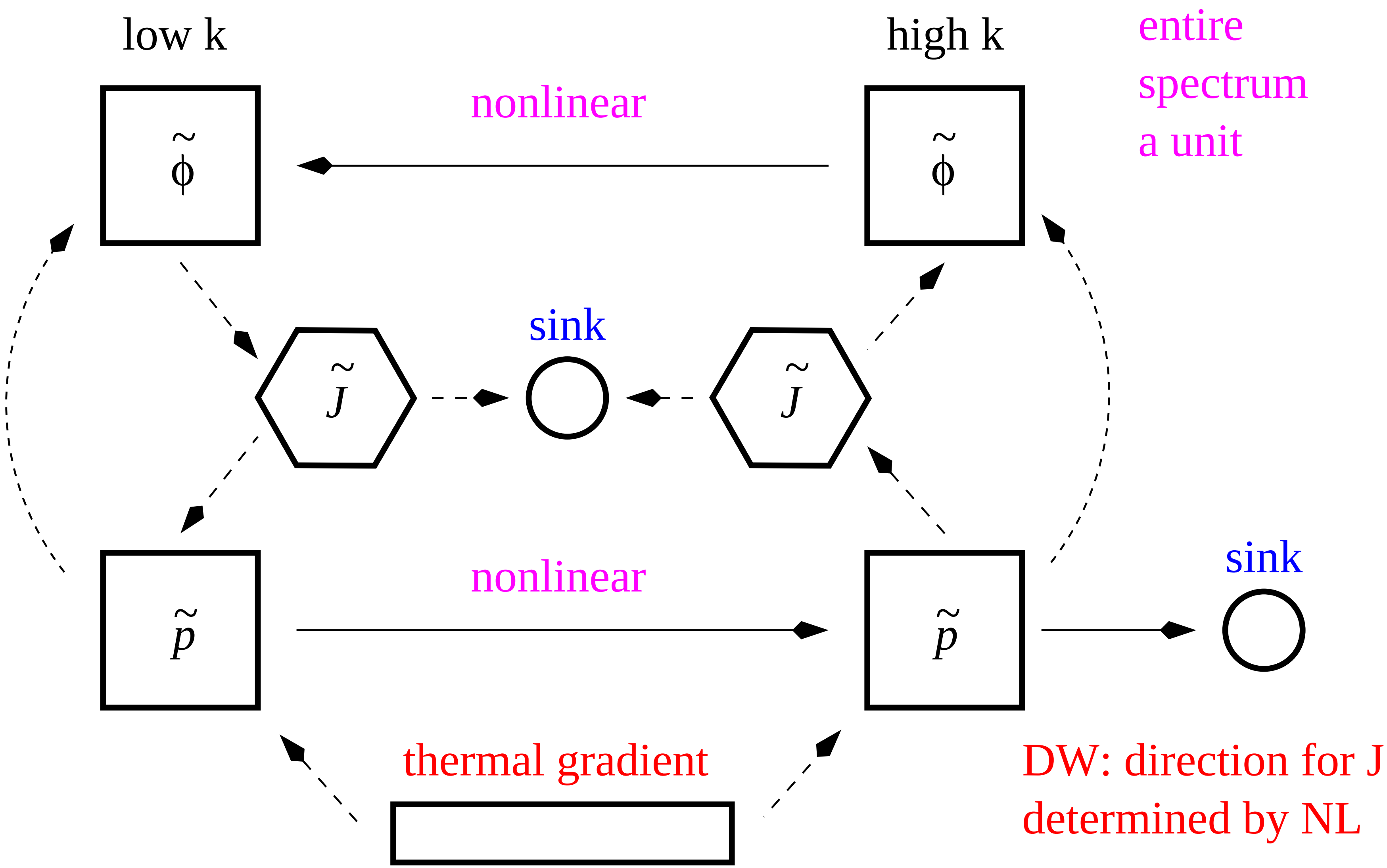
$$\frac{1}{2}\frac{\partial}{\partial t}\left(\beta_e^{-1}\left|\rho_s\nabla_\perp A_\parallel\right|^2 + \mu_e J_\parallel^2\right) + \nabla\cdot() = \frac{J_\parallel}{B}B\nabla_\parallel(p_e-\phi) - 0.51\mu_e\nu_e J_\parallel^2$$

$$\frac{1}{2}\frac{\partial}{\partial t}p_e^2 = p_e B\nabla_\parallel\frac{J_\parallel}{B} + p_e\mathcal{K}(\phi - p_e)$$

- processes in two-fluid models only
  - electron adiabatic compression $J_\parallel\nabla_\parallel p_e$
  - diamagnetic compression $p_e\mathcal{K}(p_e)$

- processes in all models: ExB compression, $p_e\mathcal{K}(\phi)$, and sound waves

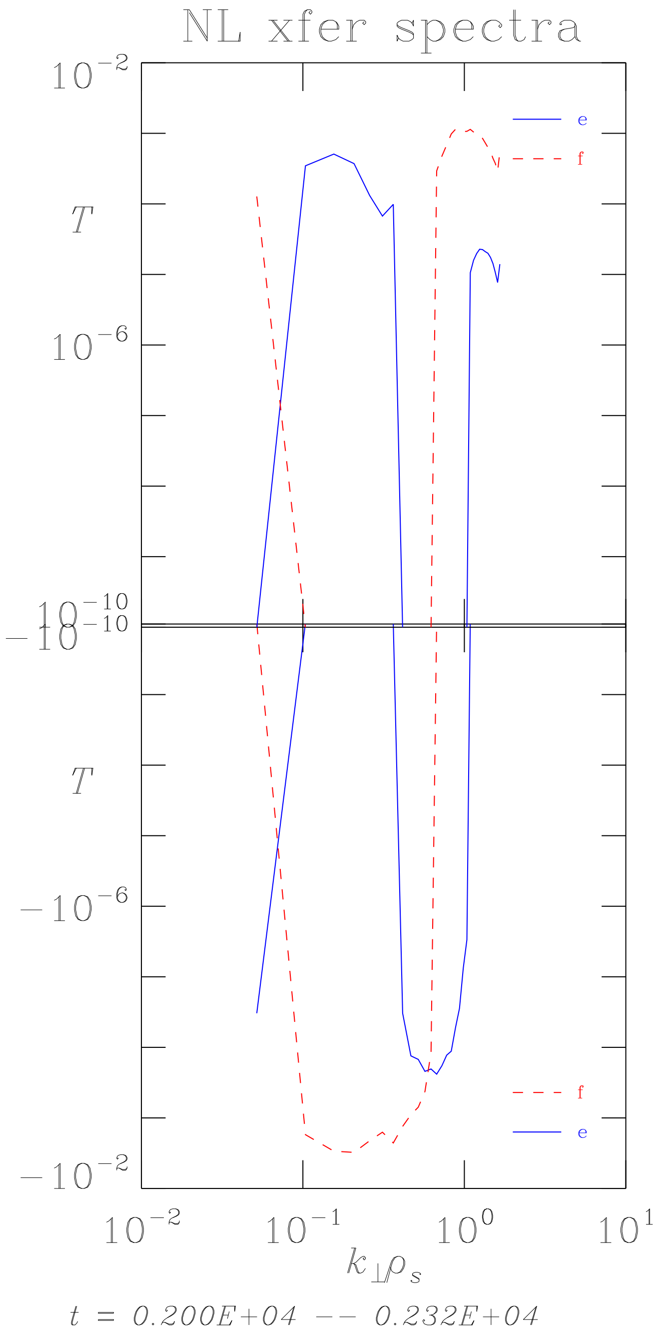
Energy Transfer: electromagnetic turbulence



(B Scott Phys Fluids B 1992, Plasma Phys Contr Fusion 1997)

(S Camargo et al Phys Plasmas 1995 and 1996)

Nonlinear Free Energy Cascade



direct cascade in delta−f entropy

energy taken out of larger scales

−−> nonlinear drive at small scales

inverse cascade in ExB energy

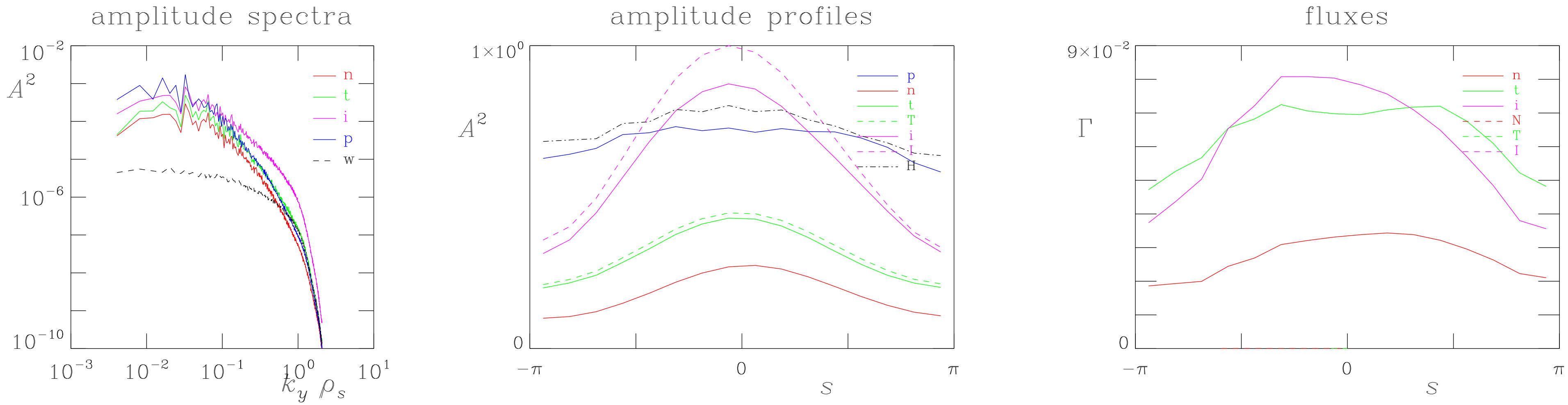
−−> nonlinear drive of long−wave MHD component

spectrum tied together

scalings are affected

turbulence signatures

- basic turbulence with finite-beta drift wave mode structure



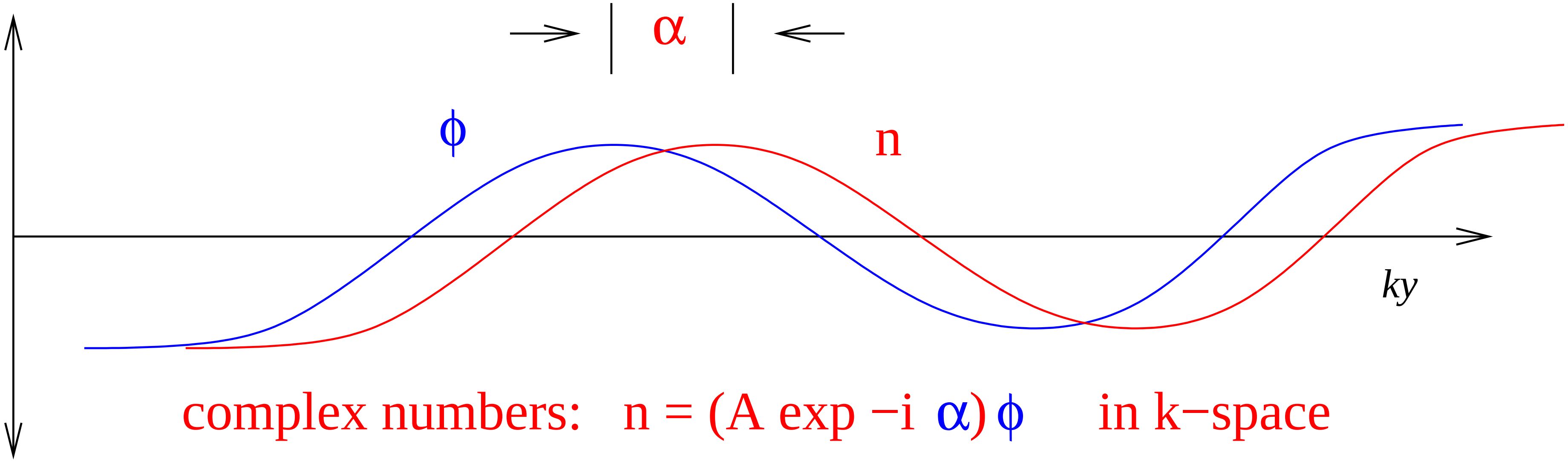
- spectra: mesoscale MHD activity, vorticity (‘w’) extends to ion gyroradius

- envelopes: ion temperature (magenta) largest fluctuation, most strongly ballooned
  ◦ potential (blue) is flat: shear-Alfvén signature, one of the dissipation channels

- fluxes: moderate, not extreme, ballooning (2 to 1 is common)

Phase Shifts and their Measurement

one dependent variable quantity leads another in the drift direction



complex numbers: $n = (A \exp -i\ \alpha)\,\phi$ in k−space

amplitude/phase are real numbers

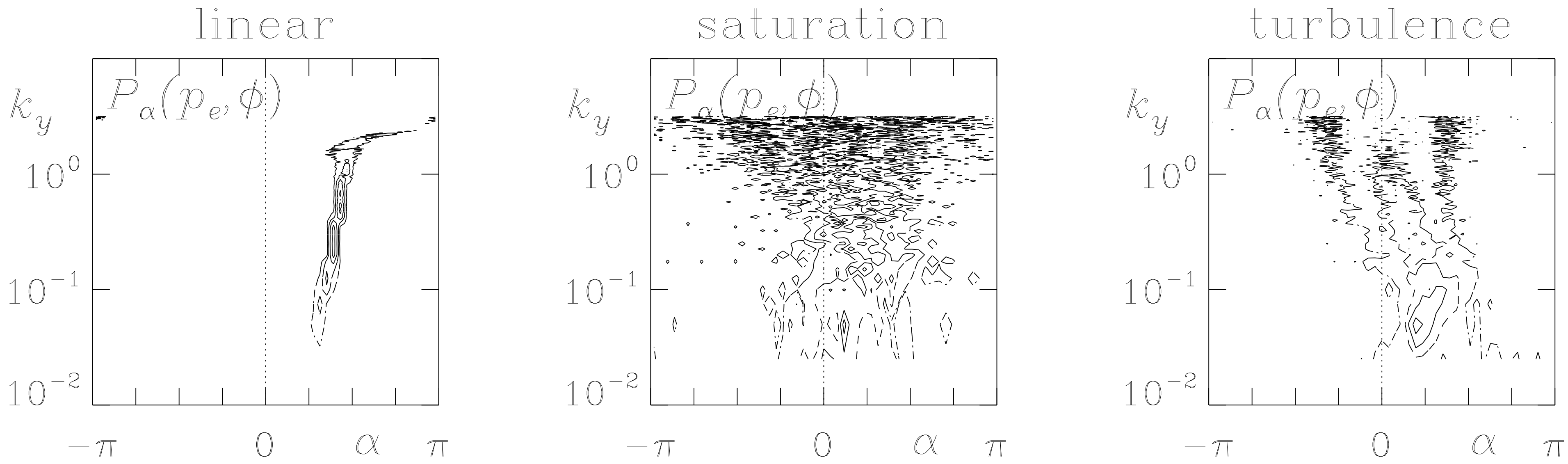
how to calculate α : $\alpha = \mathrm{Im}\log n^*\phi$

significance: a positive phase−shift implies a positive down−gradient flux

(B Scott Plasma Phys Contr Fusion 1997)

Nonlinear Transition

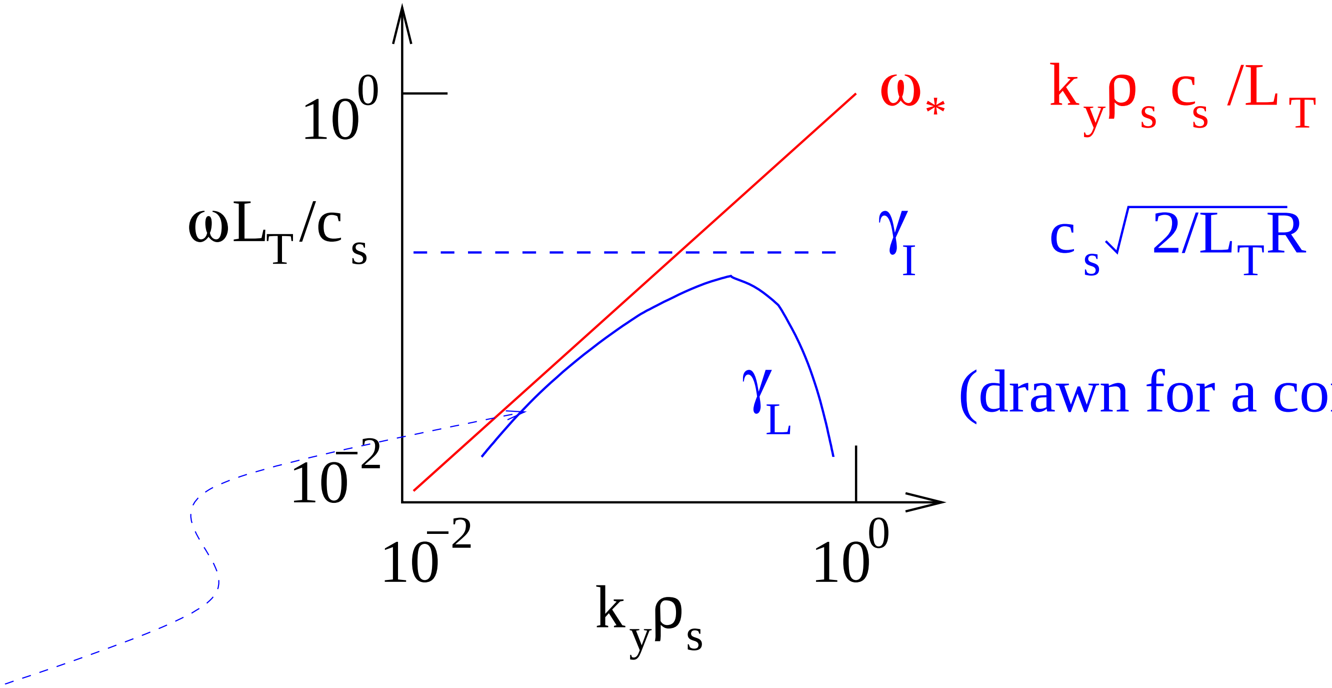
linear mode structure wiped out by turbulence after saturation



linear regime phase shift part of the eigenmode for each k_y
linear mode structure destroyed by the turbulence during saturation
mode structure at late times is the turbulence one: DW mode structure

Relevance Range for Linear Instabilities

dispersion space bounded by ideal interchange and diamagnetic rates



if the linear growth rate is below the red line then the instability is irrelevant

usually, this is not the case anywhere in the spectrum (unless: MHD threshold)

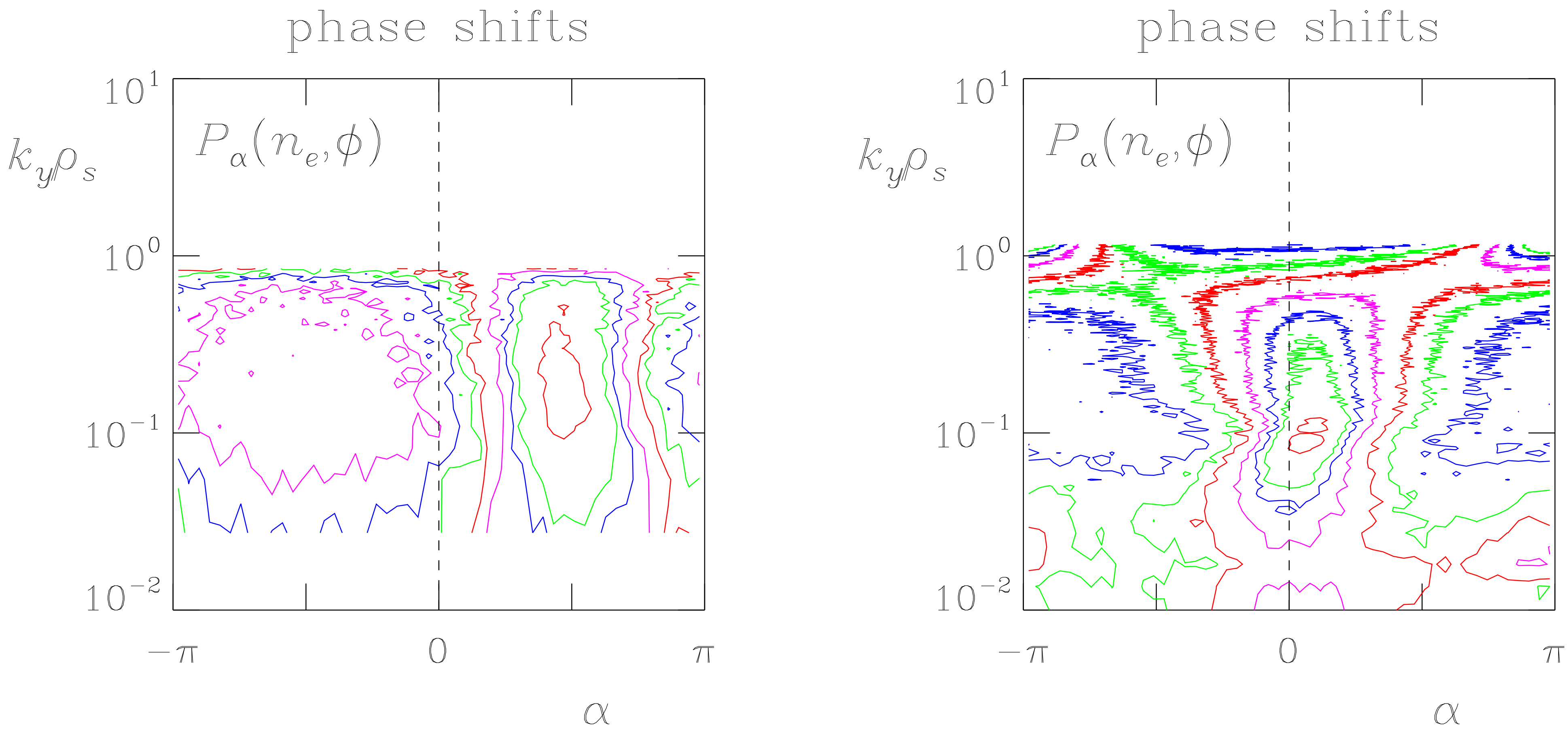
this situation is a direct consequence of very large $R/L_T >> 1$ in the edge

(B Scott New J Phys 2002, Phys Plasmas 2005)

phase shifts – RMHD versus full gyrofluid

- gyrofluid turbulence is driven by both ∇T_e and ∇T_i
  - ◦ despite the ballooned structure of T_i
  - ◦ the $n_e \leftrightarrow \phi$ phase shifts remain drift-wave like, $\alpha \gtrsim 0$

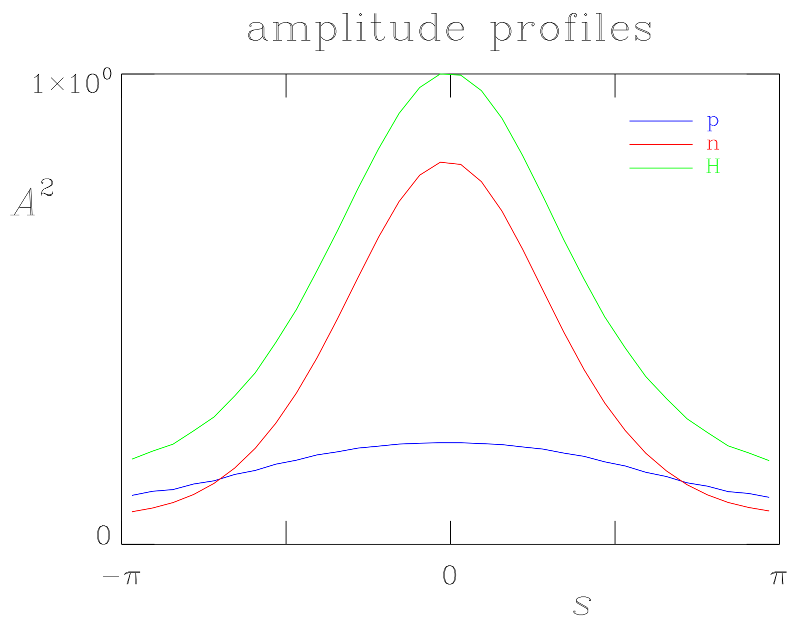
- this is **completely different from RMHD**, which has $\alpha \sim \pi/2$

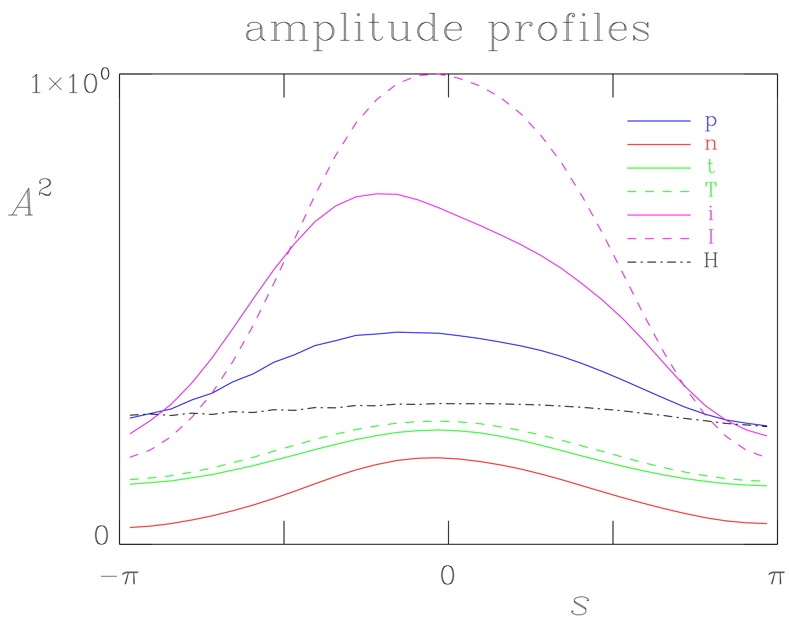


parallel structure – RMHD vs full gyrofluid

- gyrofluid turbulence is driven by both ∇T_e and ∇T_i
  - despite the ballooned structure of T_i
  - the nonadiabatic part of p_e is almost flat

- this is **completely different from RMHD**,
  - which violates its assumption that $n_e e \nabla\phi \gg \nabla p_e$ (this always happens)





summary – edge turbulence basics

- ions say ITG, electrons say DW, $\widetilde{\phi}$ says trans-MHD $\rightarrow$ **all are present/active**

- gyrofluid edge-ITG signatures:
  - T_i is largest and most ballooned ($T_{i\perp}$ a little more than $T_{i\|}$)
  - $H = n_e - \phi$ is the flattest
  - Alfvén signature: ϕ flatter than T_e which is flatter than n_e
  - nevertheless, the electron and ion ExB fluxes are comparable

- these features are why in your model for edge turbulence . . .

you need to keep $p_e \leftrightarrow J_\| \leftrightarrow \phi$ adiabatic response

you need T_i distinct from p

you need to resolve ρ_i in any computations

flow energetics in a tokamak

- main element in a tokamak: geodesic curvature
  ◦ poloidal gradient in magnetic field in perpendicular drift dynamics
  ◦ **absent in a 2D/interchange model**

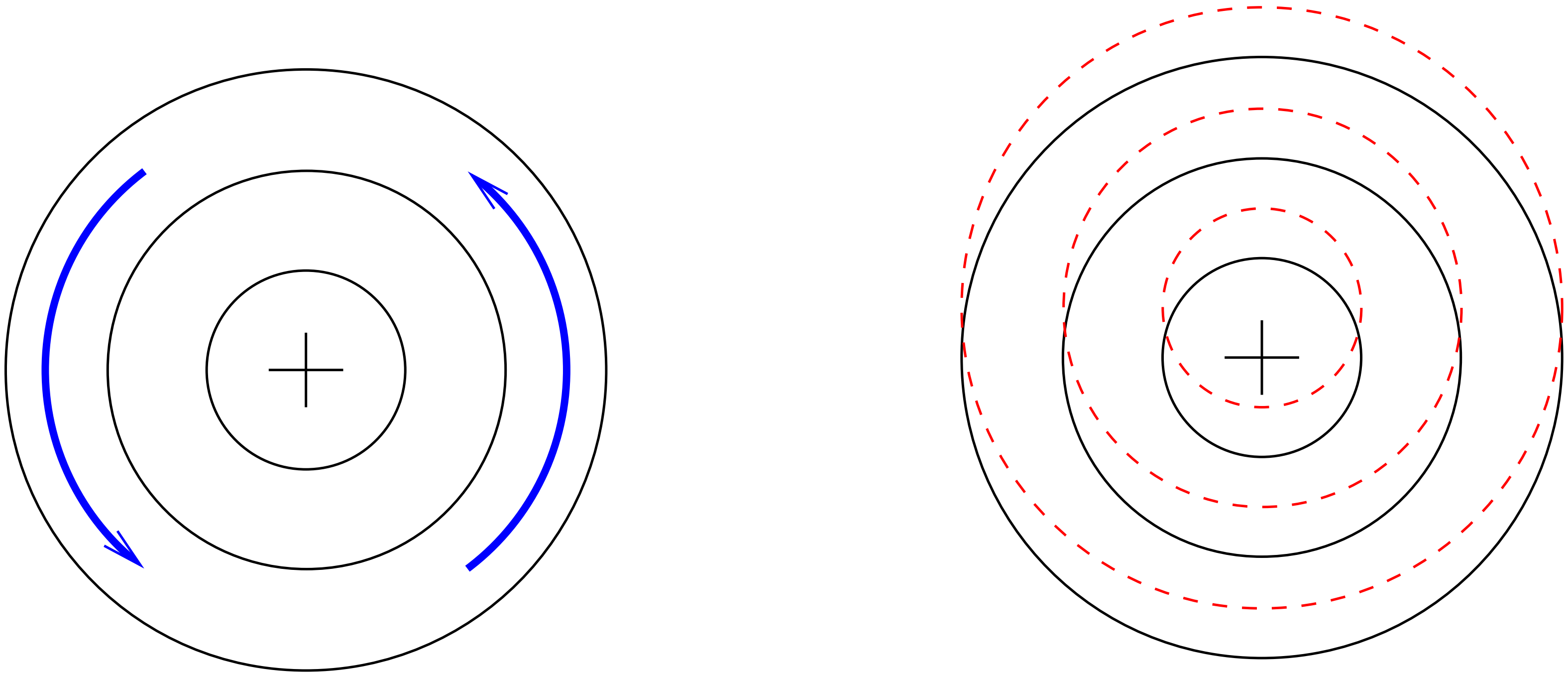
- main two-fluid element for flows:
  ◦ coupling between acoustic and Alfvén branches

- fate of zonal flow energy: transfer to dissipation channels via conservative processes
  ◦ turbulence $\rightarrow$ incoherent mixing
  ◦ adiabatic sideband compression $\rightarrow$ parallel electron dissipation

- how to do the analysis: sideband decomposition
  ◦ zonal and $\sin\theta$ components of state variables (ϖ, p_e)
  ◦ sin cos component of flux variables ($J_\|$, $u_\|$)
  ◦ keep NL only as source/sink effects via turbulence fluxes, flow stress (Γ, R_E)
  ◦ don't order $\partial/\partial t$

Zonal Flow, Toroidal Compression

(Winsor et al Phys Fl 1968, Hahm et al Plasma Phys Contr Fusion 2002, 2004)



zonal flow
$\langle \phi \rangle$

compression at top
divergence at bottom

pressure sideband
$\langle p \sin\theta \rangle$

zonal flow exchanges conservatively with pressure sideband

−−> transfer pathway, equipartition

detailed example – continuity equation

- electrons: start with

$$\frac{\partial p_e}{\partial t} + [\phi, p_e] + B\nabla_\parallel \frac{u_\parallel - J_\parallel}{B} = \mathcal{K}\left(\phi - p_e\right)$$

- zonal component: flux surface average, fsavg , annihilates linear $B\nabla_\parallel$
- the bracket terms represent turbulent flux (transport)

$$\langle [\phi, p_e] \rangle + \left\langle B\nabla_\parallel \frac{u_\parallel - J_\parallel}{B} \right\rangle = \frac{\partial}{\partial x} \langle \Gamma \rangle$$

- geodesic curvature has $\sin\theta$ component, only one to survive fsavg

$$\frac{\partial}{\partial t} \langle p_e \rangle + \frac{\partial}{\partial x} \langle \Gamma \rangle = \omega_B \frac{\partial}{\partial x} \langle (\phi - p_e) \sin\theta \rangle \qquad\qquad \omega_B = \frac{2\rho_s}{R}$$

- the sideband is the disturbance maintained by gradients and finite collisionality

detailed example – continuity sideband

- electrons: sideband component: multiply by $\sin\theta$ and then take fsavg
- integrate $\partial/\partial\theta$ in $\nabla_\parallel$ by parts under fsavg

$$\left\langle \sin\theta \nabla_\parallel u_\parallel \right\rangle = -\left\langle u_\parallel \nabla_\parallel \sin\theta \right\rangle = -k_\parallel \left\langle u_\parallel \cos\theta \right\rangle \qquad k_\parallel = \frac{L_\perp}{qR}$$

- in the curvature term approximate fsavg of $\sin^2\theta$ by $1/2$

$$\langle \sin\theta \mathcal{K}(\phi) \rangle = \omega_B \left\langle \sin^2\theta \frac{\partial\phi}{\partial x} \right\rangle = \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi \rangle$$

- resulting pressure sideband equation (treat fluxes as in zonal part)

$$\frac{\partial}{\partial t} \langle p_e \sin\theta \rangle + \frac{\partial}{\partial x} \langle \Gamma \sin\theta \rangle - k_\parallel \left\langle \left(u_\parallel - J_\parallel \right) \cos\theta \right\rangle = \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi - p_e \rangle$$

detailed example – parallel forces

- electrons: start with

$$\frac{\partial}{\partial t}\left(A_\parallel + \mu_e J_\parallel\right) + [\phi, \mu_e J_\parallel] = \nabla_\parallel (p_e - \phi) - 0.51\mu_e \nu_e J_\parallel$$

- flux variable sideband component: multiply by $\cos\theta$ and then take fsavg
  - integrate $\partial/\partial\theta$ in $\nabla_\parallel$ by parts under fsavg (watch signs)
  - the nonlinear terms in this equation are small (MHD stable regime)

$$\frac{\partial}{\partial t}\left\langle\left(A_\parallel + \mu_e J_\parallel\right)\cos\theta\right\rangle = k_\parallel \left\langle (p_e - \phi)\sin\theta\right\rangle - 0.51\mu_e\nu_e\left\langle J_\parallel \cos\theta\right\rangle$$

detailed example – parallel forces

- ions: start with

$$\mu_i \frac{\partial}{\partial t} u_\parallel + [\phi, \mu_i u_\parallel] = -\nabla_\parallel p_e + \mu_\parallel \nabla_\parallel^2 u_\parallel$$

- flux variable sideband component: multiply by $\cos\theta$ and then take fsavg
  - integrate $\partial/\partial\theta$ in $\nabla_\parallel$ by parts under fsavg (watch signs)
  - the nonlinear terms in this equation are small (subsonic regime)

$$\frac{\partial}{\partial t} \left\langle \mu_i u_\parallel \cos\theta \right\rangle = -k_\parallel \left\langle p_e \sin\theta \right\rangle - \mu_\parallel k_\parallel^2 \left\langle u_\parallel \cos\theta \right\rangle$$

detailed example – charge conservation

- charge: start with the vorticity equation

$$\frac{\partial\varpi}{\partial t} + [\phi, \varpi] = B\nabla_\parallel \frac{J_\parallel}{B} - \mathcal{K}(p_e)$$

- zonal component: treat as in the zonal continuity equation
  - the bracket term represents Reynolds/Maxwell stresses (ExB forcing)

$$\langle[\phi, \varpi]\rangle - \left\langle B\nabla_\parallel \frac{J_\parallel}{B} \right\rangle = \frac{\partial^2}{\partial x^2} \langle R_E \rangle$$

- geodesic curvature has $\sin\theta$ component, only one to survive fsavg

$$\frac{\partial}{\partial t} \langle\varpi\rangle = -\frac{\partial^2}{\partial x^2} \langle R_E \rangle - \omega_B \frac{\partial}{\partial x} \langle p_e \sin\theta \rangle$$

- the sideband is the disturbance maintained by gradients and finite collisionality

Sideband Dynamics for Flows/Currents

- zonal vorticity, pressure sideband, sound wave sideband

$$\frac{\partial}{\partial t}\left\langle \varpi \right\rangle = -\frac{\partial^2}{\partial x^2}\left\langle R_E \right\rangle - \omega_B \frac{\partial}{\partial x}\left\langle p_e \sin\theta \right\rangle$$

$$\frac{\partial}{\partial t}\left\langle p_e \sin\theta \right\rangle + \frac{\partial}{\partial x}\left\langle \Gamma \sin\theta \right\rangle = \frac{\omega_B}{2}\frac{\partial}{\partial x}\left\langle \phi \right\rangle - \frac{\omega_B}{2}\frac{\partial}{\partial x}\left\langle p_e \right\rangle + k_\parallel \left\langle u_\parallel \cos\theta \right\rangle - k_\parallel \left\langle J_\parallel \cos\theta \right\rangle$$

$$\frac{\partial}{\partial t}\mu_i \left\langle u_\parallel \cos\theta \right\rangle = -k_\parallel \left\langle p_e \sin\theta \right\rangle - \mu_\parallel k_\parallel^2 \left\langle u_\parallel \cos\theta \right\rangle$$

- Alfvén sideband, flow sideband, zonal pressure

$$\frac{\partial}{\partial t}\left\langle \left(A_\parallel + \mu_e J_\parallel\right)\cos\theta \right\rangle = k_\parallel \left\langle p_e \sin\theta \right\rangle - k_\parallel \left\langle \phi \sin\theta \right\rangle - 0.51\mu_e\nu_e \left\langle J_\parallel \cos\theta \right\rangle$$

$$\frac{\partial}{\partial t}\left\langle \varpi \sin\theta \right\rangle = -\frac{\partial^2}{\partial x^2}\left\langle R_E \sin\theta \right\rangle - k_\parallel \left\langle J_\parallel \cos\theta \right\rangle - \frac{\omega_B}{2}\frac{\partial}{\partial x}\left\langle p_e \right\rangle$$

$$\frac{\partial}{\partial t}\left\langle p_e \right\rangle + \frac{\partial}{\partial x}\left\langle \Gamma \right\rangle = \omega_B \frac{\partial}{\partial x}\left\langle \phi \sin\theta \right\rangle - \omega_B \frac{\partial}{\partial x}\left\langle p_e \sin\theta \right\rangle$$

zonal flow energetics

- the equations don't all fit on the page, but this is what you do . . .
  - here, one equation per page

- for the zonal vorticity multiply by fsavg of $-\phi$ and integrate over $d\mathcal{V}$

$$\int d\mathcal{V} \times \qquad \langle -\phi \rangle \frac{\partial}{\partial t} \langle \varpi \rangle = \langle \phi \rangle \frac{\partial^2}{\partial x^2} \langle R_E \rangle + \langle \phi \rangle \frac{\partial}{\partial x} \omega_B \langle p_e \sin\theta \rangle$$

$$\int d\mathcal{V} \times \qquad \frac{1}{2} \frac{\partial}{\partial t} \langle \rho_s \nabla_\perp \phi \rangle^2 = \langle \varpi \rangle \langle R_E \rangle - \omega_B \langle p_e \sin\theta \rangle \frac{\partial}{\partial x} \langle \phi \rangle$$

- flow energy driven by Reynolds/Maxwell stress (corr. with zonal vorticity)

- flow energy is depleted by geodesic compression

turbulence driven zonal flows saturate at low levels

pressure sideband energetics

- for the pressure sideband multiply by $2\langle p_e \sin\theta\rangle$ and integrate over $d\mathcal{V}$

$$\int d\mathcal{V} \times \qquad 2\langle p_e \sin\theta\rangle \frac{\partial}{\partial t}\langle p_e \sin\theta\rangle = \cdots$$

$$+\,\omega_B \langle p_e \sin\theta\rangle \frac{\partial}{\partial x}\langle\phi\rangle - 2\langle p_e \sin\theta\rangle \frac{\partial}{\partial x}\langle\Gamma \sin\theta\rangle - 2k_\parallel \langle p_e \sin\theta\rangle \left\langle J_\parallel \cos\theta\right\rangle$$

$$\int d\mathcal{V} \,\times \qquad \frac{\partial}{\partial t}\langle p_e \sin\theta\rangle^2 = \cdots$$

$$+\,\omega_B \langle p_e \sin\theta\rangle \frac{\partial}{\partial x}\langle\phi\rangle - 2\langle\Gamma \sin\theta\rangle \left\langle -\frac{\partial p_e}{\partial x}\sin\theta\right\rangle - 2k_\parallel \langle p_e \sin\theta\rangle \left\langle J_\parallel \cos\theta\right\rangle$$

- drive by geodesic compression (conservative transfer from zonal flow energy)
- sink by turbulent mixing and adiabatic compression . . .

turbulent mixing is one of the main sinks for zonal flow energy

Ohmic sideband energetics

- for the current sideband multiply by $2\left\langle J_\parallel \cos\theta\right\rangle$ and integrate over $d\mathcal{V}$

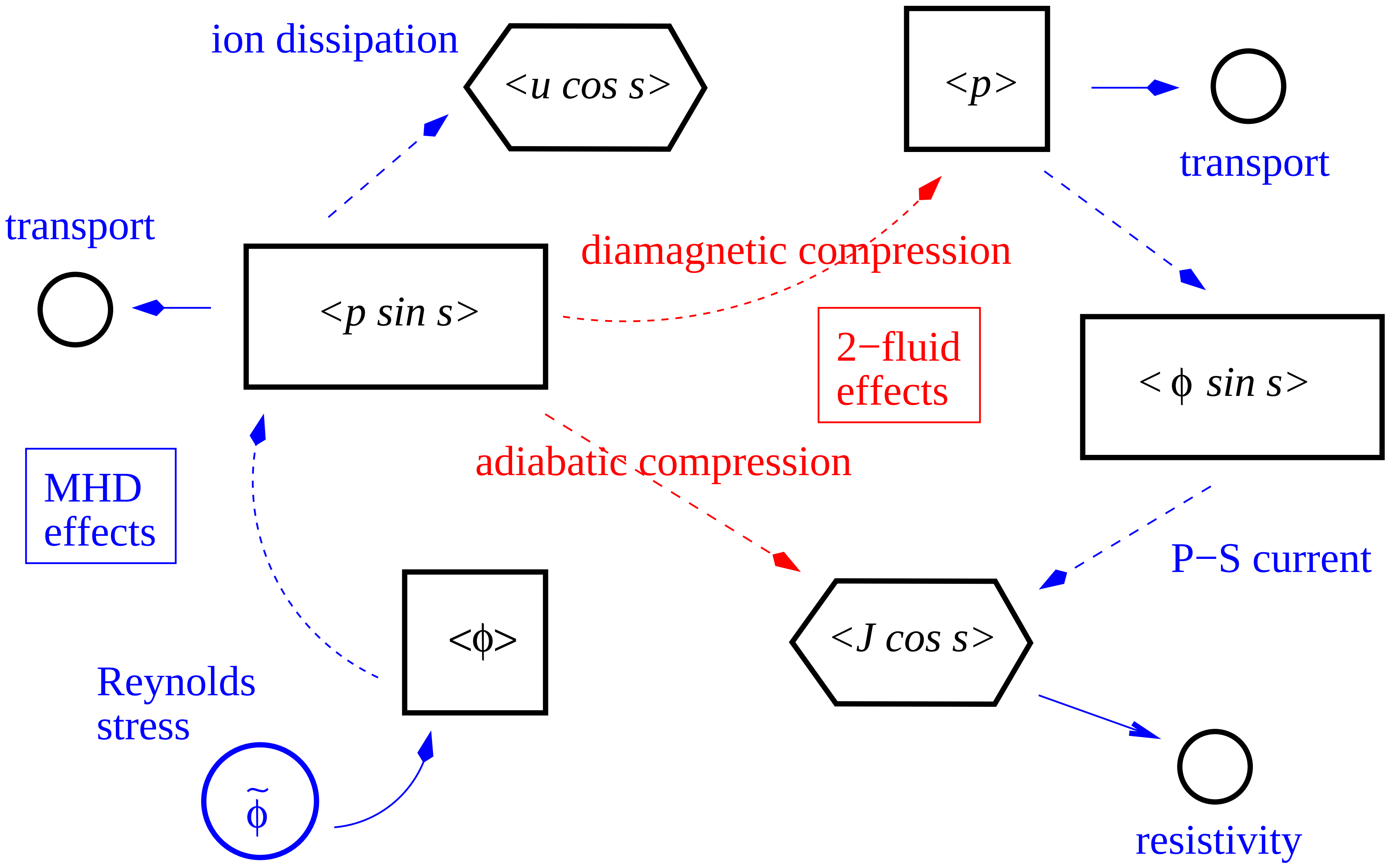
$$2\left\langle J_\parallel \cos\theta\right\rangle \frac{\partial}{\partial t}\left\langle A_\parallel \cos\theta\right\rangle = \ldots + 2k_\parallel \left\langle J_\parallel \cos\theta\right\rangle \left\langle p_e \sin\theta\right\rangle - 2\eta_\parallel \left\langle J_\parallel \cos\theta\right\rangle^2$$

$$\frac{\partial}{\partial t}\beta_e^{-1}\left\langle \rho_s(\nabla_\perp A_\parallel)\cos\theta\right\rangle^2 = \ldots + 2k_\parallel \left\langle p_e \sin\theta\right\rangle \left\langle J_\parallel \cos\theta\right\rangle - 2\eta_\parallel \left\langle J_\parallel \cos\theta\right\rangle^2$$

- magnetic and parallel electron kinetic energy (the $\mu_e J_\parallel$ term, not shown)
- drive by adiabatic compression (conservative transfer from pressure sideband energy)
  - the other process, Alfvénic compression, leads to the Pfirsch-Schlüter current (equil.)
- sink by dissipation (here, resistivity)

the sink by dissipation is the other main sink for zonal flow energy

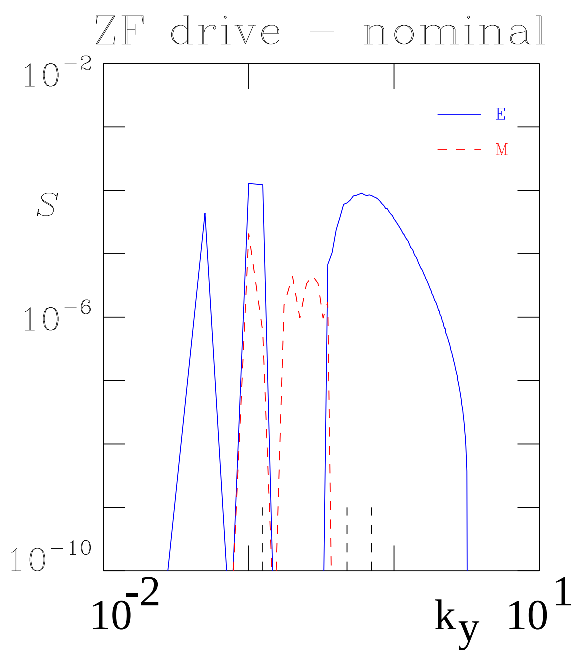
Energy Transfer: flows and currents

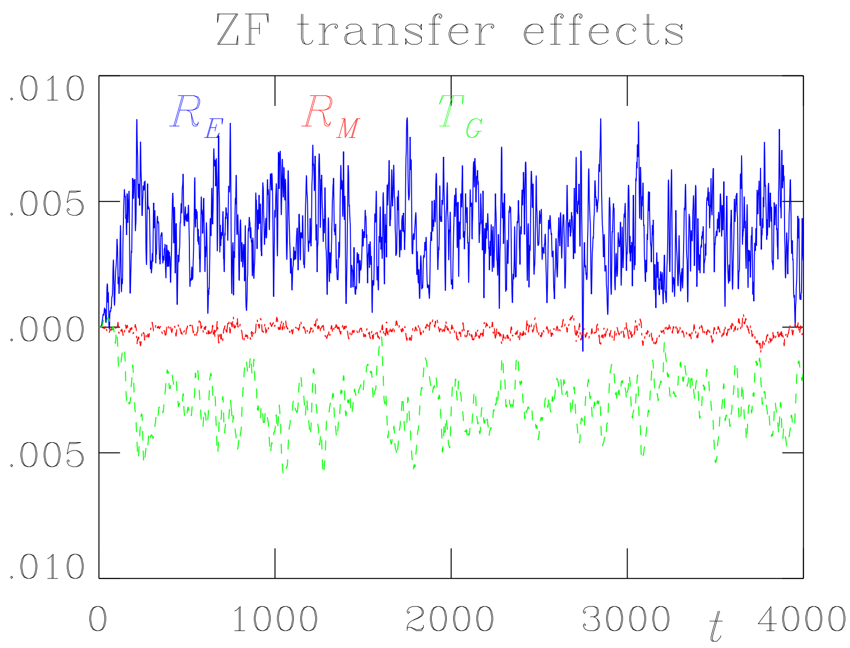


(B Scott Phys Lett A 2003, New J Phys 2005)

Coupling to Zonal Flows

turbulence regulated by flows, regulated by toroidal compression





eddy Reynolds stress --> energy transfer from turbulence to flows

turbulence moderately weakened but not suppressed

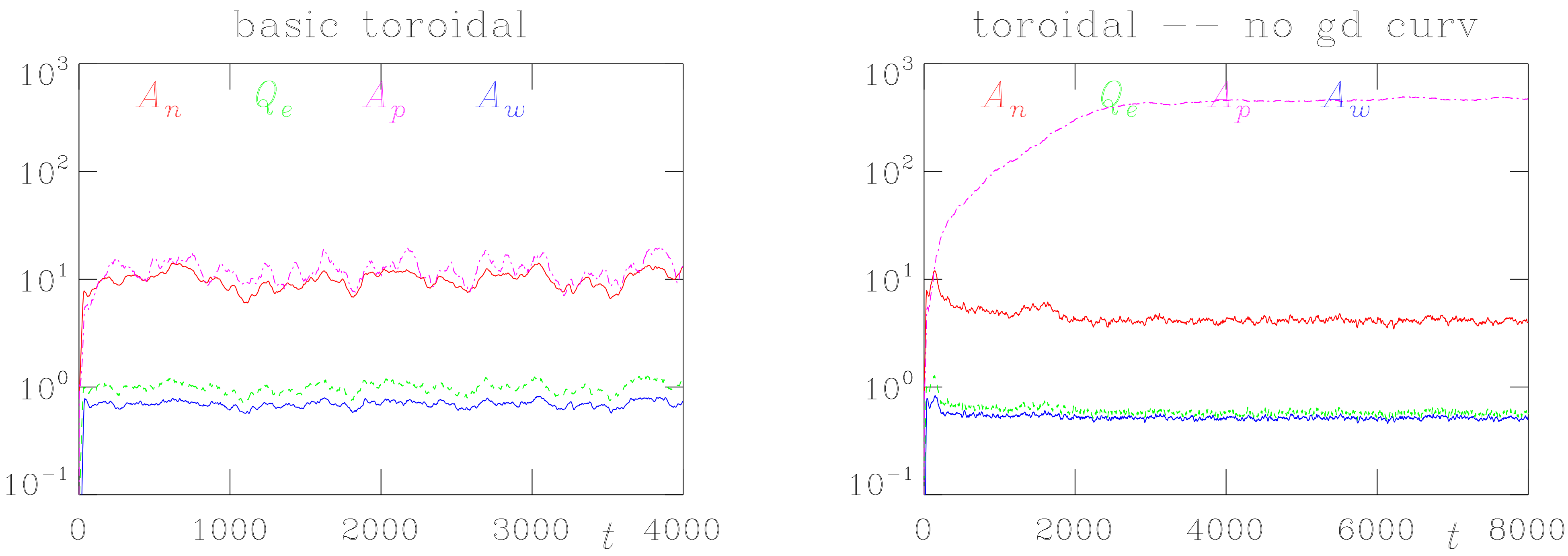
toroidal compression --> energy loss channel to pressure, turbulence

entire system in self regulated statistical equilibrium (turb, flows, mag eq)

(B Scott Phys Lett A 2003, New J Phys 2005)

Turbulence vs Geodesic Curvature

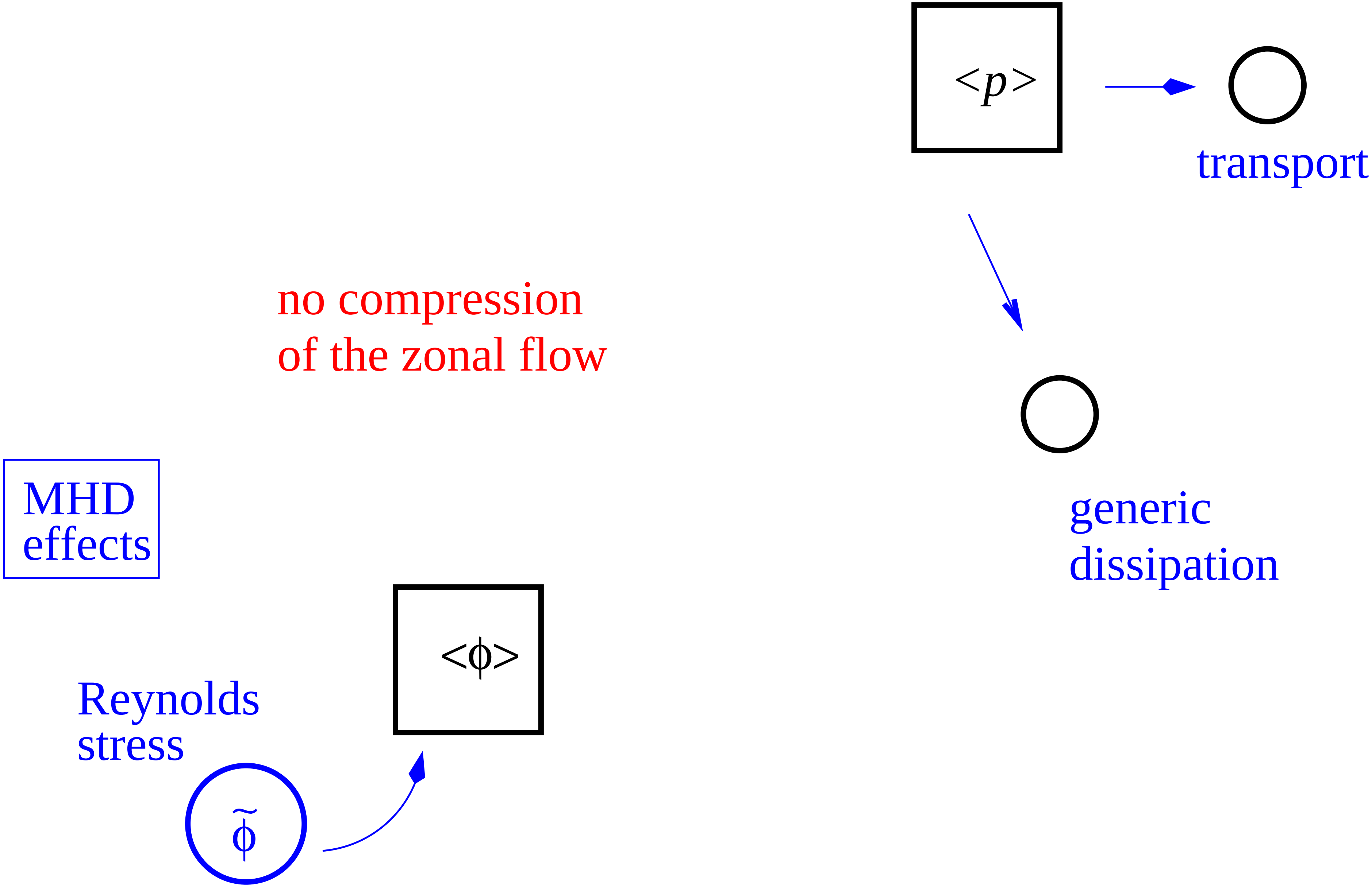
- no geodesic curvature $\Longrightarrow$ no sideband dynamics
  - details: *New J Phys* 7 (2005) 92



- self-generated flows are held down by geodesic compression . . .
  - $\Longrightarrow$ coupling back to turbulence, coupling to dissipative currents

- flows have weak effect on saturated energy, no role for predator-prey mechanism

Energy Transfer: flows and currents in 2D



(B Scott Phys Lett A 2003, New J Phys 2005)

what is gyrokinetic

- low frequency approximations, usually also low-β and small a/qR
- polarisation density, not polarisation current
- gyrocenter charge density $\leftrightarrow$ vorticity, polarisation current
- ambipolarity of **particle** charge density holds
- ambipolarity of **gyrocenter** charge density holds only in steady state $\partial/\partial t = 0$
- gauge transformation (coordinate changes, addition of pure divergences)
  ◦ addition of a pure divergence $\Longrightarrow$ integrations by parts
- equivalent to gyroaveraging over a ring orbit only at linear order
- Lagrangian/Hamiltonian support $\rightarrow$ automatic energetic consistency
  ◦ works in practice only if field equations are obtained from the same Lagrangian

what is gyrofluid

- it is a **representation** not a closure and not an ordering
- example: Hasegawa-Wakatani (fold the gradient term n_0 into n)

$$\frac{\partial n}{\partial t} + [\phi,\, n] = B\nabla_\parallel \frac{J_\parallel}{B} \qquad\qquad \frac{\partial}{\partial t}\nabla_\perp^2\phi + [\phi,\, \nabla_\perp^2\phi] = B\nabla_\parallel \frac{J_\parallel}{B}$$

subtract and define N

$$\frac{\partial N}{\partial t} + [\phi,\, N] = 0 \qquad\qquad N = n - \nabla_\perp^2\phi$$

- this is nothing more and nothing less than the simplest gyrofluid model
  ◦ equations for n and N with "polarisation" $-\nabla_\perp^2\phi = N - n$

no polarisation drift for gyrocenters, but a polarisation density

how to treat FLR

- lots of detail, but the simplest long-wavelength version is

$$\phi_G = \left(1 + \frac{\rho_i^2}{2}\nabla_\perp^2\right)\phi \qquad\qquad n_G = \left(1 + \frac{\rho_i^2}{2}\nabla_\perp^2\right)N$$

then

$$\frac{\partial N}{\partial t} + [\phi_G, N] + \cdots \qquad \text{and} \qquad -\rho_s^2\nabla_\perp^2\phi = n_G - n$$

- lots of algebra including bracket forms such as

$$\nabla_\perp^2[f,g] = \nabla\cdot[f,\nabla_\perp g] + \nabla\cdot[\nabla_\perp f, g] \qquad \nabla\cdot[f,\nabla_\perp g] = [\nabla_\perp f, \nabla_\perp g] + [f, \nabla_\perp^2 g]$$

- use the fact that $\tau_i\rho_s^2 = \rho_i^2$, grind away,
  - ∘ and recover the fluid polarisation drift divergence, label $p_i = \tau_i n$

$$\frac{\partial n}{\partial t} + [\phi, n] - \nabla\cdot\left(\frac{\partial}{\partial t} + [\phi, \;]\right)\nabla_\perp(\phi + p_i) + \cdots$$

fluid vs gyrofluid – same model

- using these methods, can show that gyrofluid FLR covers fluid nonlinear polarisation
- dissipation . . .
- large-ν_i limit of $p_{i\,\|} - p_{i\,\perp}$ represents parallel viscosity
  ◦ including heat flux crossover (arises from v-dependence of C-operator)
- large-ν limit of $q_{\|\|\|}, q_{\perp\|} \to q_\|$ covers thermal conduction, parallel thermal force
- result: thermal gyrofluid model covers reduced Braginskii in all aspects
- reference: *Phys Plasmas* **14** (2007) 112318

gyrofluid model is . . . easier to maintain computationally
has an obvious descent from underlying gyrokinetic theory
covers pedestal dynamics in transcollisional regime

gyrokinetics and neoclassics

- basic assumptions of neoclassical theory applied to the gyrokinetic equation system
  - gyrokinetic theory requires only $\rho_L \ll L_\perp$ (forced by $\Omega_E \ll \Omega_i$)
  - conventional neoclassical theory requires $\rho_L(qR/a) \ll L_\perp$

- split f into background F^M and disturbance δf arising from thermal gradients

- usually neglect FLR corrections, since $\rho_L \ll L_\perp$
  - note orbit width comes from drifts, not FLR *per se*

- resulting equation set is identical to that used in neoclassical theory

- then, the fully nonlinear set you started with can be used in computations
  - (that is, if you coded them as they stand)

basic assumption of neoclassical theory

- time scales: slow compared to relaxation dynamics, fast compared to transport

$$\frac{v_A}{qR} \quad > \quad \frac{c_s}{R} \quad > \quad \nu_i \quad > \quad \frac{\partial}{\partial t} \quad > \quad \frac{\chi_{\rm NC}}{L_\perp^2}$$

- space scales:
  - this also follows from drifts $\ll$ parallel/collisional relaxation

$$\rho_B \quad \sim \quad \rho_s \frac{qR}{a} \quad < \quad L_\perp$$

marginal but not strongly violated even in pedestal
for conventional tokamaks

neoclassical models

- drift-kinetic equation with small drift piece acting on Maxwellian $F^M(\epsilon, \mu, \psi_c)$
  - note $\partial/\partial t$ on sideband piece is neglected by the ordering

$$v_\| \nabla_\| \left(\delta f + \frac{Ze}{T} \phi F^M \right) - C(\delta f) = -\mathbf{v}_d \cdot \left(\nabla F^M + F^M \frac{Ze}{T} \nabla \phi \right)$$

- flow damping rate ν_{NC}: maximal ordering $v_\|/qR \sim \nu_i$ suborderings:
  - banana regime: $v_\|/qR > \nu_i \quad \rightarrow \quad \nu_B \quad \propto \quad \nu_i$
  - collisional regime: $v_\|/qR \quad < \quad \nu_i \quad \rightarrow \quad \nu_{PS} \quad \propto \quad \nu_i^{-1}$
  - plateau regime, often modeled with simple crossover function (WM Stacey, 1990s)

$$\nu_{NC} = \nu_B \nu_{PS} / (\nu_B + \nu_{PS})$$

- standard treatments work through moment equations
  - conserved quantities $\leftrightarrow F^M$
  - fluxes, relaxation $\leftrightarrow \delta f$

- lots of detail, endless argument … for basics refer to

  FL Hinton and RD Hazeltine, *Rev Mod Phys* **48** (1976) 239

fluid analog

- write delta-f reduced fluid equations
- do sideband analysis
- keep consequences of neoclassical ordering
  - zonal averages of conserved quantities
  - divergence balance of flux variables ($u_\parallel$, $J_\parallel$, *etc.*, no $A_\parallel$)
  - force balance of state variables (p_e, $\varpi \to \phi$, *etc.*)

detailed example – divergence balances

- electrons: start with

$$\frac{\partial p_e}{\partial t} + [\phi, p_e] + B\nabla_\| \frac{u_\| - J_\|}{B} = \mathcal{K}\left(\phi - p_e\right)$$

- state variable sideband component: multiply by $\sin\theta$ and then take fsavg
  - note $\partial/\partial t$ on sideband piece is neglected by the ordering
  - integrate $\partial/\partial\theta$ in $\nabla_\|$ by parts under fsavg
  - in the curvature term approximate fsavg of $\sin^2\theta$ by $1/2$
  - for these purposes neglect nonlinear terms

$$k_\| \left\langle \left(u_\| - J_\|\right)\cos\theta \right\rangle = -\frac{\omega_B}{2}\frac{\partial}{\partial x}\left\langle \phi - p_e \right\rangle$$

- considering ions also and separating $J_\|$ these are the divergence balances

$$k_\| \left\langle J_\| \cos\theta \right\rangle = -\frac{\omega_B}{2}\frac{\partial}{\partial x}\left\langle p_e + p_i \right\rangle \qquad k_\| \left\langle u_\| \cos\theta \right\rangle = -\frac{\omega_B}{2}\frac{\partial}{\partial x}\left\langle \phi + p_i \right\rangle$$

detailed example – parallel forces

- electrons: start with

$$\frac{\partial}{\partial t}\left(A_\parallel + \mu_e J_\parallel\right) + [\phi, \mu_e J_\parallel] = \nabla_\parallel (p_e - \phi) - 0.51\mu_e\nu_e J_\parallel$$

- flux variable sideband component: multiply by $\cos\theta$ and then take fsavg
  - note $\partial/\partial t$ on sideband piece is neglected by the ordering
  - integrate $\partial/\partial\theta$ in $\nabla_\parallel$ by parts under fsavg
  - for these purposes neglect nonlinear terms

$$k_\parallel \left\langle (p_e - \phi)\sin\theta \right\rangle = 0.51\mu_e\nu_e \left\langle J_\parallel \cos\theta \right\rangle$$

- with temperature dynamics there is more to it than this but the ideas remain

detailed example – parallel forces

- ions: start with

$$\mu_i \frac{\partial}{\partial t} u_\parallel + [\phi, \mu_i u_\parallel] = -\nabla_\parallel (p_e + p_i) + \mu_\parallel \nabla_\parallel^2 \left(u_\parallel + k q_{i\parallel} \right)$$

- flux variable sideband component: multiply by $\cos\theta$ and then take fsavg
  - note $\partial/\partial t$ on sideband piece is neglected by the ordering
  - integrate $\partial/\partial\theta$ in $\nabla_\parallel$ by parts under fsavg
  - for these purposes neglect nonlinear terms

$$k_\parallel \langle (p_e + p_i) \sin\theta \rangle = -\mu_\parallel k_\parallel^2 \left\langle \left(u_\parallel + k q_{i\parallel} \right) \cos\theta \right\rangle$$

- here we've kept enough of the temperature dynamics to get rotation effects
  - because these parallel flux variables are considered in divergence balance

$$k_\parallel \left\langle u_\parallel \cos\theta \right\rangle = -\frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi + p_i \rangle \qquad k_\parallel \left\langle q_{i\parallel} \cos\theta \right\rangle = -\frac{5}{2} \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle T_i \rangle$$

rotation – zonal charge balance

- start with (warm-ion) vorticity equation, keep polarisation, neglect nonlinearities

$$\frac{\partial}{\partial t}\rho_s^2\nabla_\perp^2\left(\phi+p_i\right)=B\nabla_\parallel\frac{J_\parallel}{B}-\mathcal{K}\left(p_e+p_i\right)$$

- zonal component

$$\frac{\partial}{\partial t}\rho_s^2\frac{\partial^2}{\partial x^2}\left\langle\phi+p_i\right\rangle=-\omega_B\frac{\partial}{\partial x}\left\langle\left(p_e+p_i\right)\sin\theta\right\rangle$$

plug in from last page, evaluate coefficients

$$\frac{\partial}{\partial t}\rho_s^2\frac{\partial^2}{\partial x^2}\left\langle\phi+p_i\right\rangle=-\frac{\mu_\parallel{\omega_B}^2}{2}\frac{\partial^2}{\partial x^2}\left\langle\left(\phi+p_i+\frac{5}{2}kT_i\right)\right\rangle$$

- this says ϕ-profile relaxes into neoclassical balance with rate $\mu_\parallel{\omega_B}^2/2$
  - ◦ nonzero fluid rotation given by kT_i piece, same (relation, damping model) as in

P Helander and DJ Sigmar, *Collisional Transport in Magnetized Plasmas* (2002)

MHD and flow equilibration processes

- the way we did it for flow energetics, *i.e.*, don't order $\partial/\partial t$
  - allow it to cover from v_A/qR to c_s/R to nu_i to transport

- displayed: isothermal version for clarity, except keeping $kq_{i\|}$ in viscosity
  - in the computations carry the entire system (12 gyrofluid equations)

- acoustic branch: zonal vorticity ϖ, sidebands for n_e and $u_\|$

- MHD branch: zonal n_e, sidebands for $J_\|$ and ϖ

- **main point:** these branches are coupled only by two-fluid processes
  - adiabatic compression among sidebands for n_e and $J_\|$ and ϕ
  - diamagnetic compression between zonal and sideband n_e
  - in the thermal version, especially important for T_i
  - why? because T_i is not constrained by Alfvén dynamics

- these processes strongly alter zonal flow relaxation
  - they are absent in the "resistive ballooning" and "2D interchange" models

- details: *New J Phys* 7 (2005) 92

Sideband Dynamics for Warm-Ion Flows

- zonal vorticity, pressure sideband, sound wave sideband

$$\frac{\partial}{\partial t}\langle\varpi\rangle = -\omega_B \frac{\partial}{\partial x}\langle(p_e + p_i)\sin\theta\rangle$$

$$\frac{\partial}{\partial t}\langle n_e \sin\theta\rangle = \frac{\omega_B}{2}\frac{\partial}{\partial x}\langle\phi - p_e\rangle + k_\parallel \left\langle u_\parallel \cos\theta\right\rangle - k_\parallel \left\langle J_\parallel \cos\theta\right\rangle$$

$$\frac{\partial}{\partial t}\mu_i \left\langle u_\parallel \cos\theta\right\rangle = -k_\parallel \langle(p_e + p_i)\sin\theta\rangle - \mu_\parallel k_\parallel^2 \left\langle\left(u_\parallel + k q_{i\parallel}\right)\cos\theta\right\rangle$$

- Alfvén sideband, flow sideband, zonal pressure

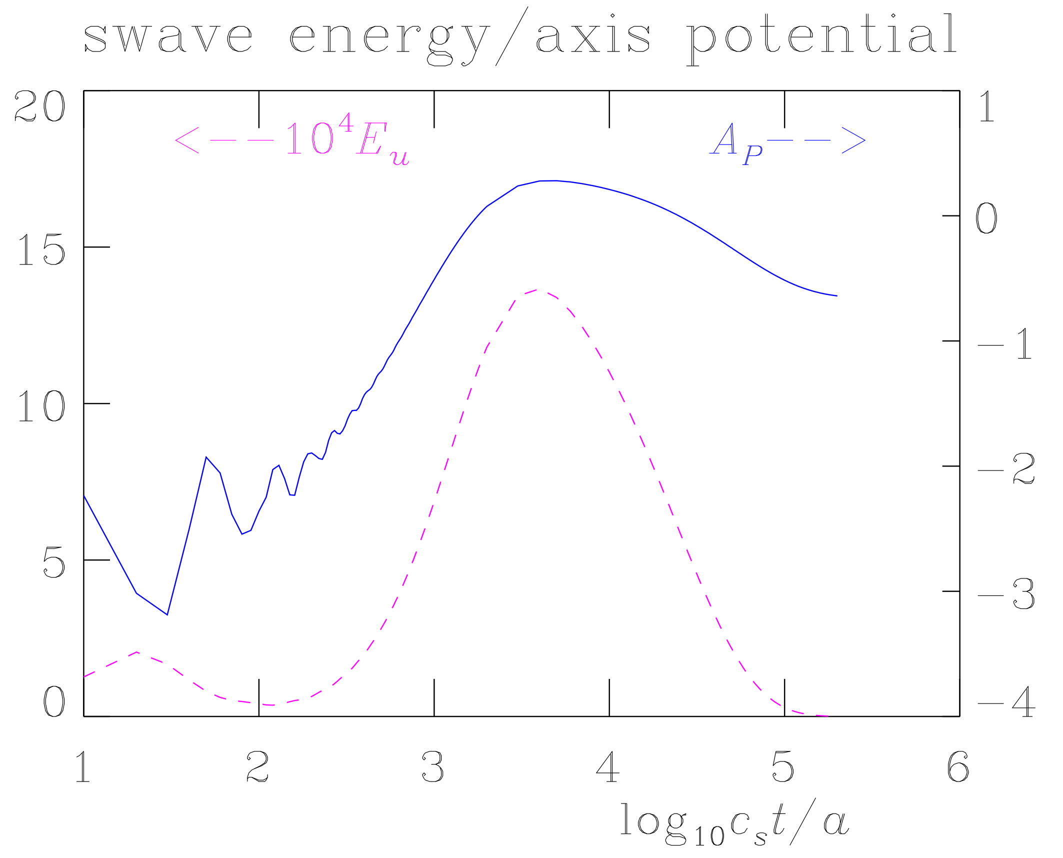
$$\frac{\partial}{\partial t}\left(\left\langle A_\parallel \cos\theta\right\rangle + \mu_e \left\langle J_\parallel \cos\theta\right\rangle\right) = k_\parallel \langle p_e \sin\theta\rangle - k_\parallel \langle\phi \sin\theta\rangle - 0.51\mu_e\nu_e \left\langle J_\parallel \cos\theta\right\rangle$$

$$\frac{\partial}{\partial t}\langle\varpi \sin\theta\rangle = -k_\parallel \left\langle J_\parallel \cos\theta\right\rangle - \frac{\omega_B}{2}\frac{\partial}{\partial x}\langle p_e + p_i\rangle$$

$$\frac{\partial}{\partial t}\langle n_e\rangle = \omega_B \frac{\partial}{\partial x}\langle\phi \sin\theta\rangle - \omega_B \frac{\partial}{\partial x}\langle p_e \sin\theta\rangle$$

flow equilibration in the fluid model

- logarithmic time axis, shows all successive phases
- rotation balances (c_s/R), relaxes (ν_i), then decays with transport
  ◦ radial electric field (A_P is axis value of ϕ), poloidal flow (E_u is energy in $u_\parallel$)



relaxation in a gyrokinetic model

- large-scale "shear-Alfvén gyrokinetics" with fields $\phi, A_\parallel$ and no gyroaveraging
  ∘ axisymmetric geometry and dynamics – a 4D (2X×2V) model

Contrib Plasma Phys **50** (2010) 228 and *Phys Plasmas* **17** (2010) 112302

- for each species (ions, electrons, $Z = \pm 1$)

$$B_\parallel^* \frac{\partial f}{\partial t} + \nabla H \cdot \frac{c}{Ze} \mathbf{b} \times \nabla f + \left(\mathbf{B} + p_z \frac{c}{Ze} \nabla \times \mathbf{b} \right) \cdot \left(\frac{\partial H}{\partial p_z} \nabla f - \frac{\partial f}{\partial p_z} \nabla H \right) = C(f)$$

- self-consistent fields

$$\sum_{\mathrm{sp}} \int d\mathcal{W} \left[Zef + \frac{1}{B_\parallel^*} \nabla \cdot B_\parallel^* \frac{fmc^2}{B^2} \nabla_\perp \phi \right] = 0$$

$$\nabla_\perp^2 A_\parallel + \frac{4\pi}{c} \sum_{\mathrm{sp}} \int d\mathcal{W} \left[\frac{Ze}{m} \left(p_z - \frac{Ze}{c} A_\parallel \right) f \right] = 0$$

what Lagrangian did we get this from

- gyrocenter Lagrangian and Hamiltonian in coordinates $\{\mathbf{R}, p_z, \mu\}$

$$L_p = \left(\frac{Ze}{c}\mathbf{A} + p_z\mathbf{b}\right)\cdot\dot{\mathbf{R}} + \frac{mc}{Ze}\,\mu\dot{\vartheta} - H$$

$$H = \frac{1}{2m}\left(p_z - \frac{Ze}{c}A_\parallel\right)^2 + \mu B + Ze\phi - \frac{mc^2}{2B^2}\left|\nabla_\perp\phi\right|^2$$

- full system Lagrangian as a field theoretical model

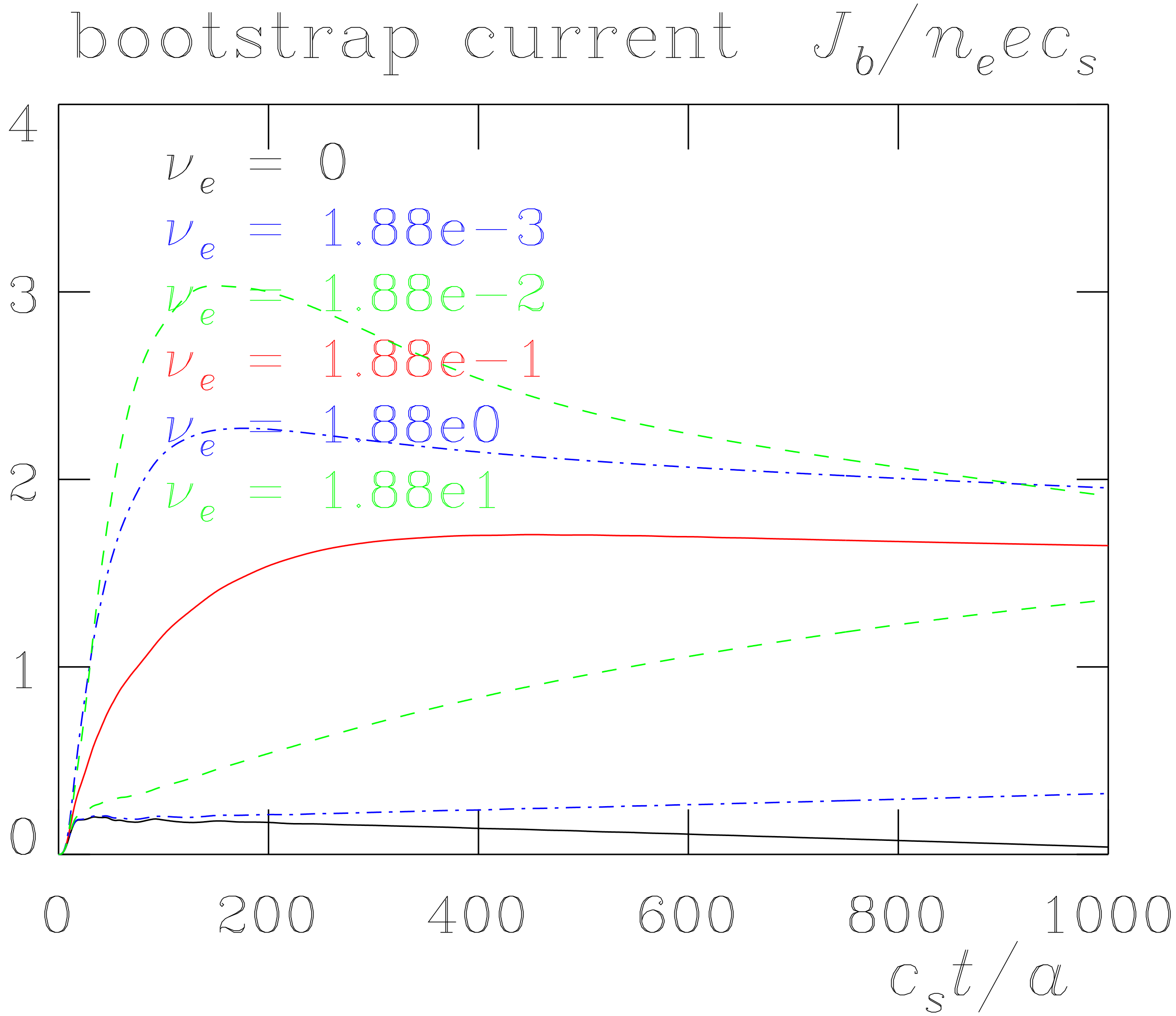
$$L = \int d\mathcal{V}\,\mathcal{L} \qquad\qquad \mathcal{L} = \sum_{\mathrm{sp}}\int d\mathcal{W}\,f\,L_p - \frac{1}{8\pi R^2}\left|\nabla_\perp\left(\psi + A_\parallel R\right)\right|^2$$

- integration over space, $d\mathcal{V}$, and over velocity space, $d\mathcal{W}$
  - in this case $\mathbf{b} = R\nabla\varphi$ and $B_\parallel^* = B$

- vary coordinates (eqn for f), and field variables (eqs for ϕ, $A_\parallel$)

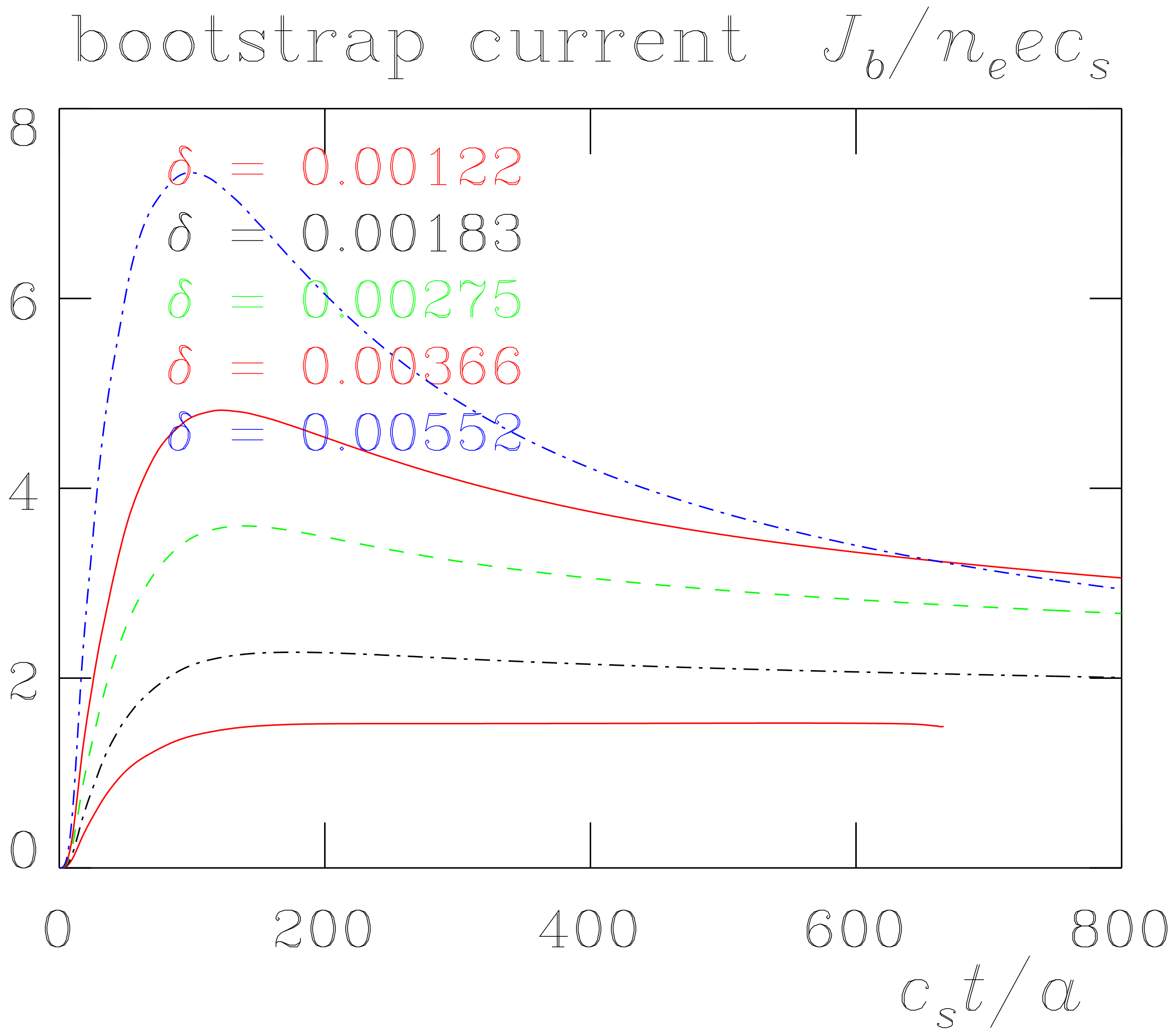
bootstrap current equilibration (collisions)

FEFI 4D, Edge Base Case, nominal $\nu_e a/c_s = 1.88$



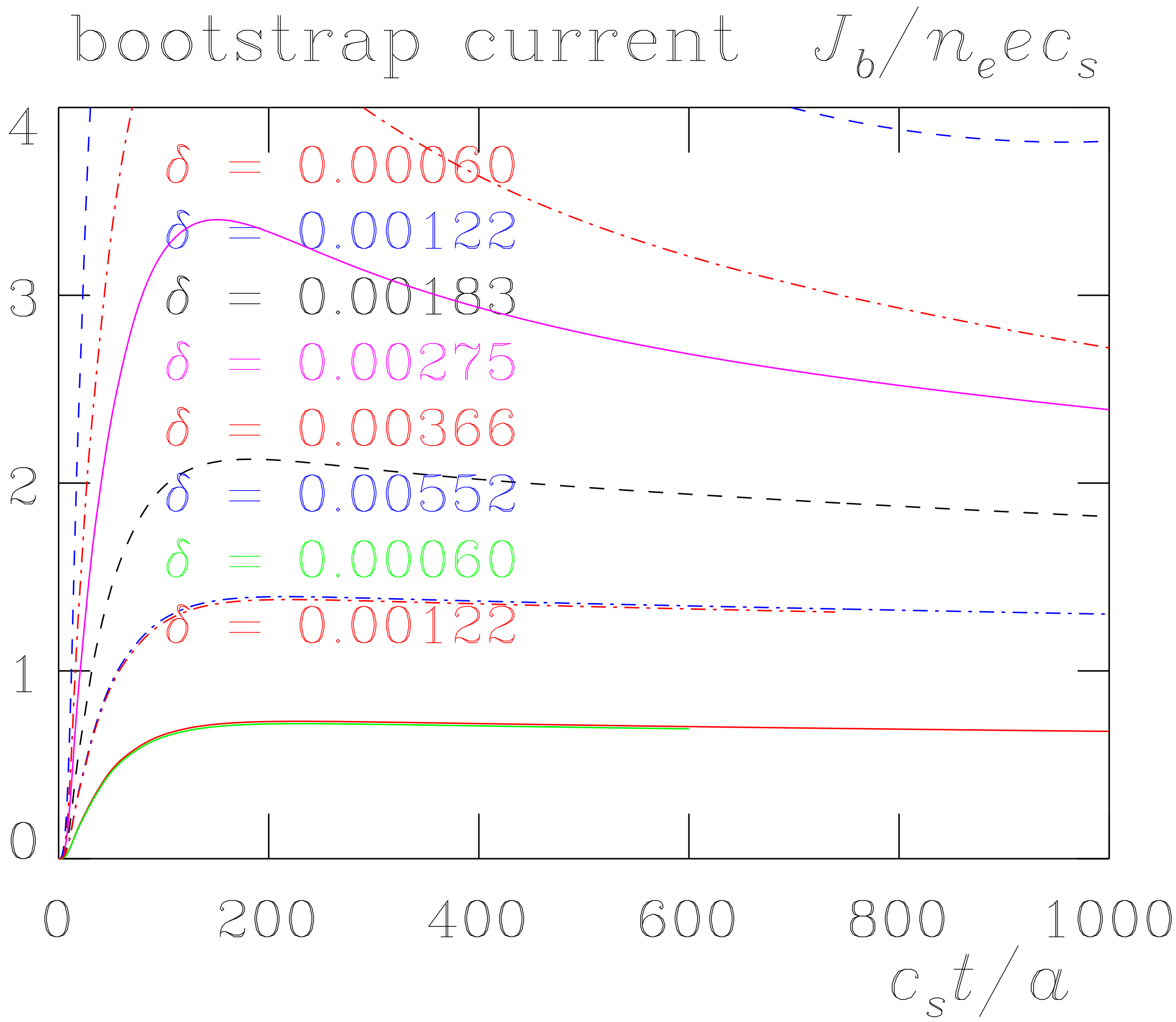
bootstrap current equilibration (rho-star)

FEFI 4D, Edge Base Case, nominal $\rho_s/a = 1.83 \times 10^{-3}$



bootstrap current equilibration (rho-star)

FEFI 4D, Edge Base Case, more ITER-relevant, to $\rho_s/a = 6 \times 10^{-4}$



squeezed orbit regime in ITER

- KC Shaing's squeezed-orbit regime requires $L_\perp \quad < \quad \rho_B$

  KC Shaing and RD Hazeltine, *Phys Fluids B* **4** (1992) 2547

- orbit squeezed by $\partial^2\phi/\partial x^2$ defined by width of constant-H curve in RZ-plane
  ∘ orbit is a 4D curve with 3 coordinates fixed (ϵ, μ, ψ_c)

- in equilibrium state, scale of ϕ tied to $L_\perp$
  ∘ but $L_\perp$ is limited by MHD stability (given fixed qR/a *etc.*)

- by definition, $\rho_s/L_\perp \lesssim a/qR$ should then be enough $\quad \rightarrow \quad$ neoclassical transport
  ∘ previous slides: only very small $\delta \lesssim 10^{-3}$ reached relaxed neoclassical state

- this was my experience running these cases: for a squeezed-regime start, transport reduced gradients before the finite ν_i could establish a steady neoclassical state

- it is hard to envisage this regime in the face of transport simultaneously occurring

- nevertheless definitive results not yet in, and it remains an interesting topic

pedestal width – models

- EPED: PB Snyder *et al*, *Phys Plasmas* **16** (2009) 056118
  - simple version sets gradient to MHD boundary and width via KBM stability
  - KBM (kinetic ballooning) is a core mode which limits the pedestal top
  - EPED represents current data sets very well
  - let's say it works for JET-size and smaller, at least

- alternative: use experience that local fluxtube models do not show H-mode transport
  - local requirement: small $\rho_s/L_\perp$
  - hence conjecture that $L_\perp/\rho_s$ has to be below some limit in the pedestal
  - evaluate parameters at pedestal halfway point, with $L_\perp = |\nabla \log T_e|^{-1}$

- the values for AUG #17151 are $L_\perp = 3\,\mathrm{cm}$ and $T = 360\,\mathrm{eV}$ and $B = 2\,\mathrm{T}$
  - this gives $L_\perp = 24\rho_s$ (if you want sharper $L_\perp$ then it's even fewer ρ_s)

- suppose we say $L_\perp$ must be below $32\rho_s$ to achieve H-mode
  - result: optimistic for AUG, approximate for JET, but very pessimistic for ITER

- suppose we (EPED or other) say in ITER you'll have $> 64\rho_s$
  - then: local conditions are felt by the turbulence within the pedestal
  - then: $\rightarrow$ no H-mode

local models don't make H mode

- all the above physics is present
  - ◦ it is a 3D model in tokamak geometry, correct boundary conditions (global consistency, periodicity constraints, no radial periodicity or ballooning)
  - ◦ 12-moment (e, i) electromagnetic gyrofluid model
  - ◦ two species (e, i) electromagnetic gyrokinetic model
  - ◦ flow effects included (both zonal flow and equilibrium flow, neoclassics)

- always find smooth monotonic rise of flux with parameters, *e.g.*, ...
  - ◦ gradient of β
  - ◦ collisionality
  - ◦ rho-star $(\rho_s/L_\perp)$ $\leftrightarrow$ system size

Edge Core Transition Power Ramp

- model: GEM, 8 flux tubes, spaced at normalised volume radius values

$$r_a = \{ \quad 0.55 \quad 0.61 \quad 0.67 \quad 0.73 \quad 0.79 \quad 0.85 \quad 0.91 \quad 0.97 \quad \}$$

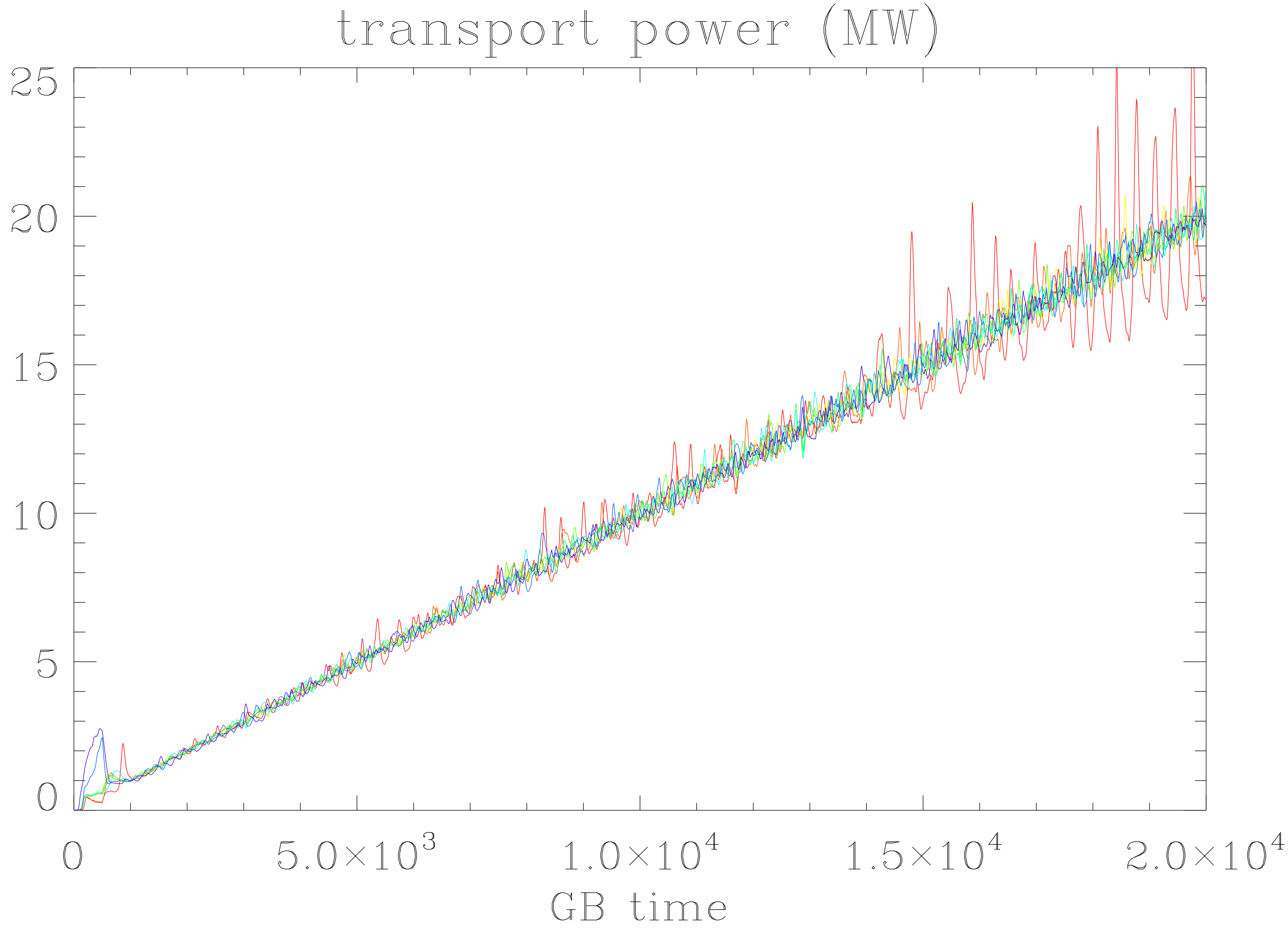
- T and ∇T for $T_e = T_i$ adjusted to get flux times sfc $\approx$ given input power
  ◦ it is an optimisation scheme, not a transport model

- GEM is formulated for all parameters, but lacks trapped electrons
  ◦ physics is found to stay in EM/NL ITG plus MHD regime anyway

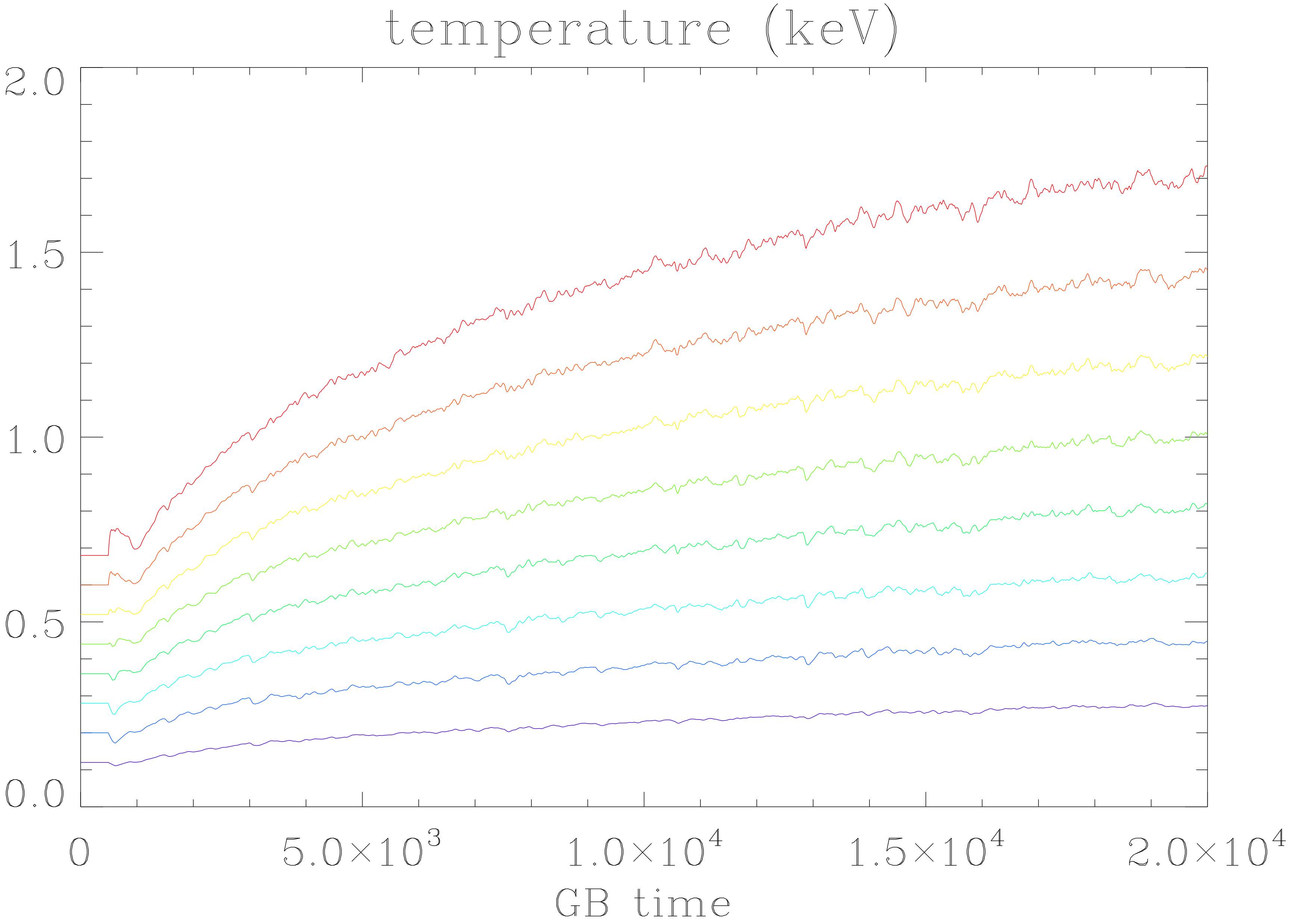
- model is AUG-sized, profiles for q, n_e, and T given with LCFS values fixed

- time traces and profiles at several times
  ◦ sweep: $P = 1\,\mathrm{MW}$ ramped after $t = 1000\tau_{GB}$ to $20\,\mathrm{MW}$ at $t = 20 \times 10^3$

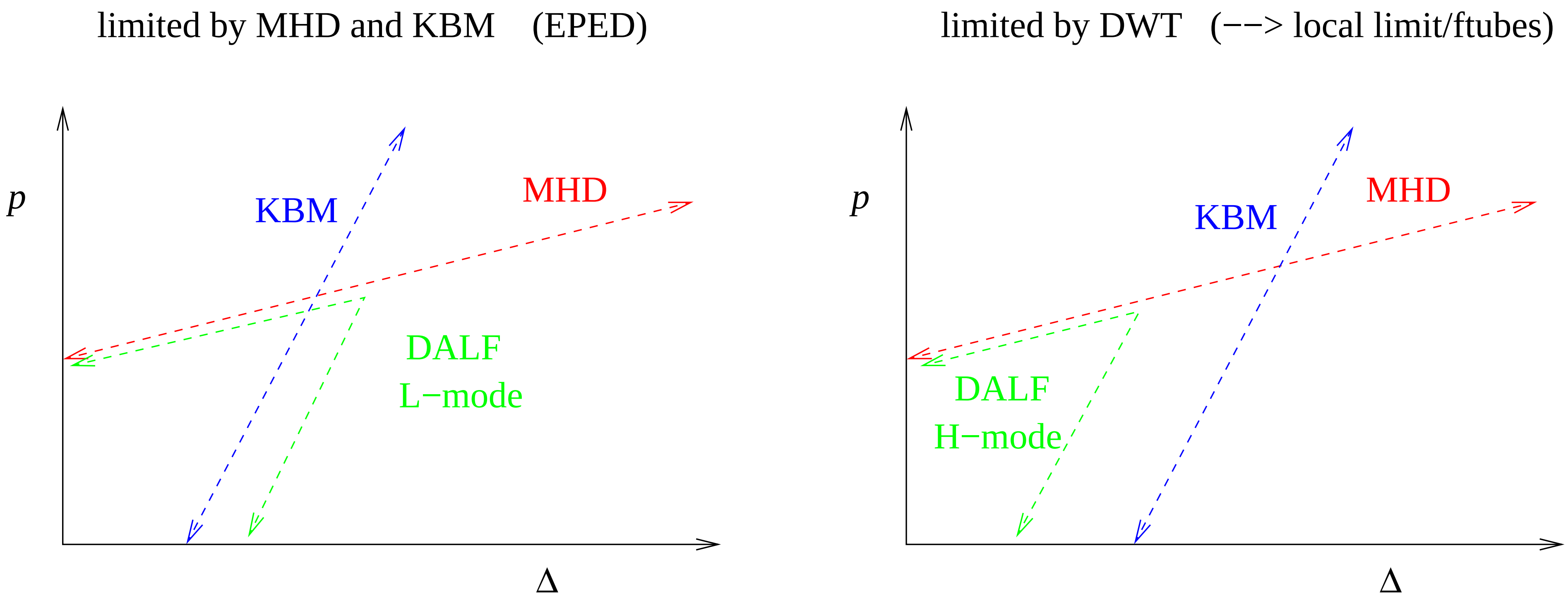
- GEM: B Scott Phys Plasmas 12 (2005) 102307 and PPCF 48 (2006) B277 power sweep to 20 MW



power sweep to 20 MW



Two Regimes of Edge Pedestal Width Limitation



it is possible that new nonlinear physics
enters to additionally limit the ITER pedestal

Edge/Pedestal – several physical processes

- we didn't cover them all, just the basics present in any reasonable model
  ◦ energetic coupling between ExB and thermal energy defines the dynamics
  ◦ not only turbulence, but also ELMs (which saturate on turbulence they generate)
  ◦ not only that but also flow energetics, coupling to resistivity through currents
  ◦ not only that but also equilibrium relaxation which involves diamagnetic compression

- gyrokinetic study of neoclassical flows is very relevant
  ◦ remember: neoclassical *process*, not "neoclassical theory"

- several external processes which may turn out to be decisive
  ◦ coupling across the LCFS (separatrix) to the SOL
  ◦ sources, *e.g.*, penetration of neutrals into the edge
  ◦ turbulence/MHD energy avalanching down from the core

- despite PR claims, no reasonable L-H transition model exists (process not known)

lots of opportunity for new people to make a mark think outside the box, stay grounded but independent you don't have to follow self-styled gurus